SAMPLE QUESTIONS AND SOLUTIONS

for Primary Level

GEMS- PRIMARY

THE ASSOCIATION
OF
MATHEMATICS TEACHERS OF INDIA

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AND
SOLUTIONS

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Primary Level

GEMS - PRIMARY

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BETWEEN US

Dear reader,

Due to persistent demand from parents, this reference material for primary children is brought out. It is done in record time to cater to the aspirants for the year 2009 itself.

The mathematics curriculum transaction in the normal class room has become so mechanical that the children are not helped to think beyond the minimum requirement, mostly memory oriented. Critical, creative thinking, analysis of data, interpreting them logically, applying in various ways the concepts introduced, are essential traits for any discerning individual these days. The earlier we introduce these to the children the better will be their overall performance.

With these ideas in mind, our colleagues Sri. G. Gnanasundaram, Sri. V. Sundaramurthy, revised our question papers from 2004 to 2007 for primary level, ably scrutinized with supply of even modified solutions by Ku. R. VIjayalakshmi to bring out this book. Sri G. Narayanan of Pagesmith comprehended our urge for expediting and completed the typesetting so early, unimaginable at that M/S. M.K. Graphics & Souri Printers also followed suit.

We hope and trust that the parents and children will find this book useful. We do welcome your suggestions to improve upon the material in subsequent editions.

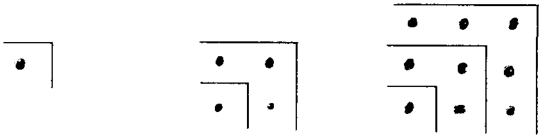
With kind regards / best wishes,

Yours sincerely,

(M.MAHADEVAN) 11.07.2009

SCREENING TEST

1. Look at the following dot diagram



This pattern continues. The value of $1 + 3 + 5 + \cdots$ up to 100 terms is the number of dots shown in the

(A) 100^{th} diagram and the number of dots present in it is 1000
(B) 1000^{th} diagram and the number of dots present in it is 10,000
(C) 100^{th} diagram and the number of dots present in it is 10,000
(D) 1000^{th} diagram and the number of dots present in it is 1000

Solution: In the first diagram there is one dot $1 = 1^2$.
1 is the first odd number $2 \times 1 - 1 = 1$.

In the second diagram there are $1 + 3 = 4$ dots $4 = 2^2$.
3 is the second odd number $3 = 2 \times 2 - 1$.

In the third diagram there are $1 + 3 + 5 = 9$ dots $9 = 3^2$.
5 is the third odd number $5 = 2 \times 3 - 1$.

In the 100^{th} diagram there are

$$\begin{aligned} 1 + 3 + 5 + \cdots + \text{ upto } 100 \text{ terms} \\ &= 1 + 3 + 5 + \cdots + (2 \times 100 - 1) \\ &= 1 + 3 + 5 + \cdots + 199 = 100^2 = 10000. \end{aligned}$$

That is sum of the first n odd numbers $= n^2$. In the above problem, we should find the sum of the first 100 odd numbers $= 100^2$.

$$\begin{aligned}(1+3+5+\cdots+99) &= (1+2+3+\cdots+200)\\ &\quad -(2+4+6+\cdots+200))\\ &= \frac{200\times 201}{2} - 2(1+2+3+\cdots+100)\\ &= 100\times 201 - \frac{2\times 100\times 101}{2}\\ &= 100\times 201 - \frac{100}{101}\\ &= 100(201-101) = 100\times 100.\end{aligned}$$

Answer: (C)

2. Look at the rows of numbers shown below

1st row :	$\boxed{1}$	$1 = \frac{1\times 2}{2}$
2nd row :	$2\ \boxed{3}$	$3 = \frac{2\times 3}{2}$
3rd row :	$4\ 5\ \boxed{6}$	$6 = \frac{3\times 4}{2}$
4th row :	$7\ 8\ 9\ \boxed{10}$	$10 = \frac{4\times 5}{2}$ and so on $\cdots$

The first number in the 50^{th} row is

(A) 1275 (B) 1224 (C) 1276 (D) 1226

Solution: The first number in the fiftieth row is the successor of the last number in the forty ninth now, namely, $\frac{49\times 50}{2} = 1225$. So, the first number in the fiftieth now is 1226.

Answer:(D)

3. In the sequence 1, 22, 333, $\cdots$ 10101010101010101010, 1111111111111111111111, $\cdots$, the sum of the digits in the 200^{th} term is

(A) 200 (B) 400 (C) 600 (D) 40000

Solution: The two hundredth member of the given sequence is the six hundred digited number.

$$200 \quad 200 \quad 200 \cdots 200$$

(200 written 200 times successively) sum of the digits of this number

$$\begin{aligned} &= (2+0+0)+(2+0+0)+\cdots 200 \text{ times} \\ &= 2 \times 200 = 400 \end{aligned}$$

Answer : (B)

4. How many two digit numbers greater than 10 are there, which are divisible by 2 and 5 but not by 4 or 25?

(A) 3 (B) 12 (C) 5 (D) 2

Solution: The numbers divisible by 2 and by 5 are multiples of 10. Such two digited numbers greater than 10 are $20, 30, \cdots, 90$. Of these 20,40,60 and 80 are divisible by 4 while 50 is divisible by 25. Thus the required answer is $8-4-1=3$.

Answer: (A)

5. The number of 3 digit even numbers that can be written using the digits 0,3,6 without repetition is

(A) 6 (B) 3 (C) 4 (D) 2

Solution: Even numbers of the type required have 0 or 6 in the units place Hundred's place cannot have 0. Thus the required numbers are only three, viz, 360,630 and 306. Answer: (B)

6. In the sequence of numbers $1, 2, 11, 22, 111, 222, \cdots$ the sum of the digits in the 999^{th} term is

(A) 999 (B) 1998 (C) 500 (D) 1000

Solution: We observe that odd term of the sequence have all digits 1 and even terms have all digits 2: First term has one 1, the third term i.e., the second odd number term has two 5^{th}

term or the third odd number term has three ones. 7th term is the 4th odd number, and have four ones.

It can be observed that n^{th} odd number is $(2n - 1)$.

Or in other words $2n - 1$ is the n^{th} odd number.

999th term $= 2 \times 500 - 1$ is the 500th (i.e., $\frac{999+1}{2} = 500$) odd number and hence has two hundred ones. The sum of its digits is also 500. Answer: (C)

7. When 1000 single digit non-zero numbers are added, the units place is 5. The maximum carry over in this case is

   (A) 495 (B) 895 (C) 899 (D) 995

   Solution: When all the thousand numbers are 9 we get the maximum sum of 9000 so that the unit's place has 0 with carry over as 900. But the unit's, place has 5. So one of the numbers must be 5 less than 9; i.e., 4 (or 999 nines and one four). Then the carry over is 899. Answer: (C)

8. You can write the number 1 using 5 and 7 and by addition and subtraction as $5+5+5-7-7=1$ (or) $7+7+7-5-5-5-5=1$ and so on. But using 3 fives and 2 sevens is the best way as we are using totally 5 numbers, whereas in the second example, we use 7 numbers. Using the above method, if 1 is written using the digits 2's and 5's only, the minimum number of times 2's and 5's are used is

   (A) three 2's and one 5 (B) three 5's and seven 2's
   (C) thirteen 2's and five 5's (D) two 2's and one 5

   Solution: There are many possibilities to write 1. viz.
   1. $(2+2+2)-5$, 2. $5-(2+2)$,
   3. $(5+5+5)-(2+2+2+2+2+2+2)$.

Also Note that:

(a) $6-5, = 16-15, = 26-25 \cdots = 1$.

(b) $5-4, = 15-14, = 25-24 \cdots = 1$.

Of these two equalities, the first ones have smallest possible numbers

$6-5 = 2+2+2-5$ and $5-4 = 5-(2+2)$ all the other forms have more number of 2^s and more number of 5^s etc.

The minimum number of twos and fives are used in $5-(2+2)$ i.e., two 2^s and one five. Answer: (D)

9. $4ab5$ is a four digit number divisible by 55 where a, b are unknown digits. Then $b-a$ is

(A) 1 (B) 4 (C) 5 (D) 0

Solution: 55 divides 4 ab 5. As $55 = 5 \times 11$ we see that 11 divides 4ab5. This means that $(4+b)-(a+5)$ or $[b-a-1]$ is divisible by 11. As a and b can be only $0, 1, 2, \cdots, 9$ it follows that $b-a-1=0$ or $b-a=1$. Answer: (A)

10. In the Fee - Vee land, the numbers are written as follows

↑	↙	↘	↗	↖	↙ ↓	↙ ↙	···	↘ ↗
0	1	2	3	4	5	6		13

then ↙ ↖ represents

(A) 7 (B) 9 (C) 14 (D) 21

Solution: Following the given pattern, the ↙ ↖ represents one five and one four i.e the decimal nine. Answer: (B)

11. The sum of the reciprocals of all the divisors of 6 is

(A) 1 (B) 2 (C) less than 2 (D) greater than 2

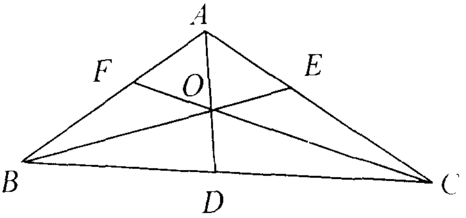
Solution: Divisors of 6 are 1,2,3 and 6.

$$\begin{aligned} \text{Sum of their reciprocals} &= \tfrac{1}{1} + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{6} \\ &= 2 \end{aligned}$$

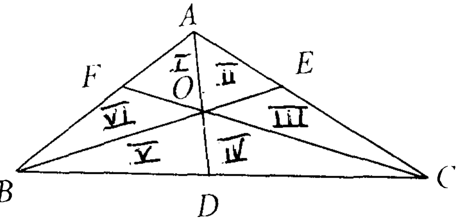
Answer: (B)

12. In the adjoining figure, the number of triangles formed is

(A) 6 (B) 7 (C) 10 (D) 16



Solution: AD, BE, CF divide the triangle ABC into six blocks I,II,$\cdots$,VI as shown in the figure. Number of triangles with just one block is equal to 6.
Number of triangle formed by two blocks taken together is 3 ((I,VI),(II,III) and (IV,V)).

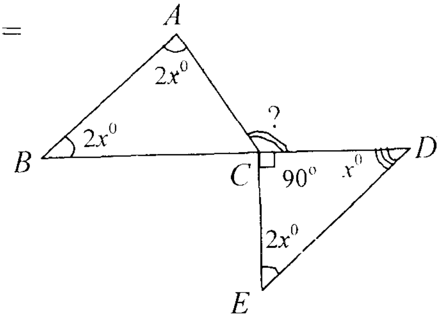


Number of triangles formed by three blocks taken together is 6 ((I,II,III) (II,III,IV) (III,IV,V), (IV,V,VI), (V,VI,I) and (VI,I,II)). No triangle can be formed by taking 4 or 5 blocks together. By taking all the six blocks together we get one triangle, namely. ΔABC. Thus the number of triangles got $= 6 + 3 + 6 + 1 = 16$.

Answer: (D)

13. In the figure shown B, C, D lie on the same line. $m\angle ECD = 90^\circ$

$$\begin{aligned} m[2\angle CDE] &= m\angle CED \\ &= m\angle BAC \\ &= m\angle ABC \end{aligned}$$



The value of $m\angle ACD$ is

(A) 120° (B) 150° (C) 210° (D) 240°

Solution: From ΔCDE, $x^\circ + 2x^\circ + 90^\circ = 180^\circ$ or $x^\circ = 30^\circ$.

$$\begin{aligned} m\angle ACD &= m\angle ABC + m\angle BAC \\ &= 2x^\circ + 2x^\circ = 4x^\circ = 4 \times 30^\circ \\ &= 120^\circ. \end{aligned}$$

Answer : (A)

14. If distinct numbers are replaced for distinct letters in the following subtraction,

$$\begin{array}{r} \text{FOUR} \\ -\ \text{ONE} \\ \hline \text{TWO} \\ \hline \end{array}$$

then the values of F and T are given by

(A) $F = 1, T = 9$ (B) $F = 1, T = 8$

(C) $F = 1$ and T is any single digit other than 1

(D) F and T cannot be determined.

Solution: Writing the problem as addition, we have

$$\begin{array}{r} \text{ONE} \\ +\ \text{TWO} \\ \hline \text{FOUR} \\ \hline \end{array}$$

F is clearly 1 because the maximum of the sum of 2 three digit numbers can be only 1998. The addition in the hundred's place gives

$O + T +$ carry over from ten's place, say $C = 10F + O$.

As $F = 1$, cancelling out 0, we have $T + C = 10$(ten)

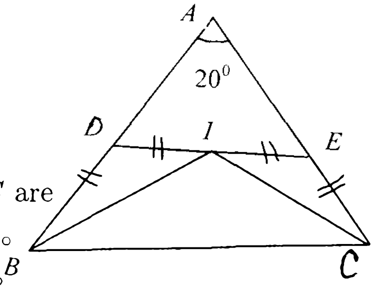
As C can be only 0 or 1 and $C + T = 10$,

we have necessarily $C = 1$and $T = 9$.

Thus $F = 1, T = 9$. Answer: (A)

15. ΔABC is an isosceles triangle with $m\angle A = 20^\circ$ and $AB = AC$. D and E are points on AB and AC such that $AD = AE$. I is the midpoint of the segment DE.

If $BD = ID$, then the angles of ΔIBC are



(A) $110^\circ, 35^\circ, 35^\circ$ (B) $100^\circ, 40^\circ, 40^\circ$
(C) $80^\circ, 50^\circ, 50^\circ$ (D) $90^\circ, 45^\circ, 45^\circ$

Solution: As ΔADE is isosceles,

$$m\angle ADE = \tfrac{1}{2}(180^0 - 20^0) = 80^0$$
$$\therefore\ m\angle BDI = m\angle CEI = 180^0 - 80^0 = 100^0$$

As $BD = DI, m\angle DBI = m\angle DIB = \frac{1}{2}(180^0 - 100^0) = 40^\circ$

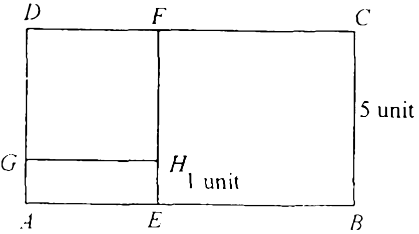
Similarly, $m\angle ICE = m\angle EIC = 40^0$.

$$\text{As } AB = AC, m\angle ABC = m\angle ACB = \tfrac{1}{2}(180^0 - 20^0)$$
$$= 80^0$$
$$\therefore\ m\angle IBC = m\angle ICB = 80^0 - 40^0 = 40^0$$

Also $m\angle BIC = 180^0 - 40^0 - 40^0 = 100^0$

Thus angles of ΔIBC are $100^0, 40^0, 40^0$ Answer: (B)

16. In the figure given $BCFE$, $DFHG$ are squares, $BC = 5$ units, $HE = 1$ unit, the length and breadth of the rectangle $ABCD$ are



(A) 8 units and 5 units (B) 5 units and 10 units
(C) 5 units and 7 units (D) 9 units and 5 units

Solution: $BCFE$ and $DFHG$ are squares.

$GH = FH = FE - HE = CB - HE = 5 - 1 = 4$ units.

$\therefore AE = GH = 4,\ EB = BC = 5$ units.
$\therefore AB = AE + EB = 4 + 5 = 9$ units.
$\therefore$ The length and breadth of the rectangle are 9 units and 5 units. Answer: (D)

17. If $!5 = 4+6-5, !12 = 11+13-12$ and $!23 = 22+24-23$, then what is the value of $!40+!41+!42+!43+!44+\cdots+!49+!50$?

(A) 505 (B) 495 (C) 455 (D) 465

Solution: From the examples given. we find that

$$!n = (n-1) + (n+1) - n = n.$$

$$\therefore \text{The required sum} = 40 + 41 + \cdots + 50 = \tfrac{11}{2}(40+50) = 495$$

Answer: (B)

18. The number of pairs of two digit square numbers, the sum or difference of which are also square numbers is

(A) 0 (B) 1 (C) 2 (D) 3

Solution: Two digit square numbers are 16,25,36,49,64 and 81.
of these $25 - 16 = 9 = 3^2$ and $36 + 64 = 100 = 10^2$.

Answer: (C)

19. $(a-1)^2 + (b-2)^2 + (c-3)^2 + (d-4)^2 = 0$.
Then $a \times b \times c \times d + 1$ is

(A) 0^2 (B) 10^2 (C) 5^2 (D) $1^2 + 2^2 + 3^2 + 4^2 + 1$

Solution: Sum of squared real quantities can be zero only if each summand is zero. Therefore,

$$a - 1 = 0;\ b - 2 = 0;\ c - 3 = 0 \text{ and } d - 4 = 0$$
$$\text{or } a = 1,\ b = 2,\ c = 3 \text{ and } d = 4.$$

So $abcd + 1 = 1 \times 2 \times 3 \times 4 + 1 = 25 \quad 5^2$. Answer: (C)

20. The adjacent table defines the operation $\star$; e.g. $b \star c = a$, $d \star a = a$ etc.

$\star$	a	b	c	d
a	b	c	d	a
b	c	d	a	b
c	d	a	b	c
d	a	b	c	d

If $b \star x = d$ then $(x \star c) \star a$ is

(A) a (B) b (C) c (D) d

Solution: From the table $b \star x = d$ implies $x = b$.
Therefore,

$$\begin{aligned}(x \star c) \star a &= (b \star c) \star a \\ &= a \star a = b\end{aligned}$$

Answer: (B)

21. The date of the 100th day of any year that is not a leap year is

(A) April 10 (B) April 2 (C) April 7 (D) April 9

Solution: In a non-leap year, the number of days elapsed upto the end of March is $31 + 28 + 31 = 90$. Hence the hundredth day will be April 10. Answer: (A)

22. The number of two digit numbers of the form aa, with the same digits 'a' having exactly four divisors is

(A) 2 (B) 4 (C) 6 (D) 8

Solution: The numbers of the form aa are

$$11, 22, 33, 44, 55, 66, 77, 88, 99$$

All these are divisible by 11. Only 11 (prime number) will have 2 divisors. The numbers $11 \times 2 = 22$, $11 \times 3 = 33$, $11 \times 5 = 55$, and $11 \times 7 = 77$ will each have exactly 4 divisors. The remaining four will have more than 4 divisors. Answer: (B)

Note: If $N = p_1^{a_1} p_2^{a_2} p_3^{a_3} \cdots p_n^{a_n}$ is the prime factorization of N, number of divisors of N is $(a_1 + 1)(a_2 + 1) \cdots (a_n + 1)$ where p_i are prime numbers.

23. The number of prime numbers dividing 2005 is

(A) 4 (B) 3 (C) 2 (D) 1

Solution: $2005 = 5 \times 401$. Divisors of 2005 are $1, 5, 401, 2005$ out of which 1 is neither prime nor composite and 2005 is composite. So the two prime divisors of 2005 are 5 and 401.

Answer: (C)

24. A 2005 digit number has all its digits the same. If this number is divided by 11, then the remainder is

(A) 0 (B) 10 (C) either 0 or 10 (D) neither 0 nor 10

Solution: Let a be the digit $0 < a \leq 9$, repeated 2005 times forming the given number. There are $\frac{2005+1}{2} = 1003$ digits in odd places and the remaining 1002 are in even places from the left. So, the difference between the sum of digits in odd positions and the sum of digits in even positions is $1003a - 1002a = a$.

This is the remainder when the number is divided by 11. But a is any integer $0 < a \leq 9$. Answer: (D)

25. The number of five digit numbers that could be written using the digits 1 and 2 only and such that each of 1 and 2 is used at least once is

(A) 12 (B) 14 (C) 32 (D) 30

Solution: Total number of five digit numbers using 1 and 2 alone is $2^5 = 32$. Out of these there are only two numbers which have only one digit. viz., 11111 and 22222. Hence the required number is $32 - 2 = 30$. Answer: (D)

Note: Given m non zero digits to form n digit numbers where $n > m$ using each of m digits at least once in $m^n - m$. In the present case $m = 2, n = 5$. $\therefore$ Required number is $2^5 - 2 = 32 - 2 = 30$. Answer: (D)

26. Given that a, b, c and d are natural numbers and that $a = bcd,\ b = cda,\ d = abc$ then $(a + b + c + d)^2$is

(A) 16 (B) 8 (C) 2 (D) 1

Solution: From $a = bcd$ and $b = cda$, we get

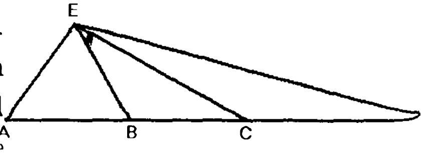
$$ab = bcd.cda = abc^2d^2 \text{ or } c^2d^2 = 1.$$

Similarly $a^2c^2 = 1$ and $b^2c^2 = 1$.

$$\therefore \frac{a^2c^2}{c^2d^2} = \frac{a^2}{d^2} = \frac{1}{1} = 1; \frac{a^2}{b^2} = 1 \text{ and } \frac{c^2}{d^2} = 1.$$

In other words, a, b, c, d being natural numbers
$a = d = b = c = 1$.
Hence $(a + b + c + d)^2 = 4^2 = 16$. Answer: (A)

27. A, B, C and D are points on a line. E is a point outside this line. Given that $AE = BE = AB = BC$ and $CE = CD$, we find that the measure of $\angle DEA$ is



(A) 90° (B) 105° (C) 120° (D) 150°

Solution: $AB = BE = AE \Rightarrow \Delta ABE$ is equilateral. $\therefore$ $\angle ABE = 60^\circ = \angle AEB$. $\therefore$ $\angle EBC = 120^\circ$. Since ΔBEC is isosceles, $\angle BEC = \angle BCE = 30^\circ$.

Also $EC = CD \Rightarrow \angle DEC = 15^\circ$ $\therefore$ $\angle DEA = \angle DEC + \angle CEB + \angle BEA = 15^\circ + 30^\circ + 60^\circ = 105^\circ$. Answer: (B)

28. The 25th term in the sequence (1,2), (2,3), (3,5), (4,7), (5,11), (6,13), $\cdots$ is

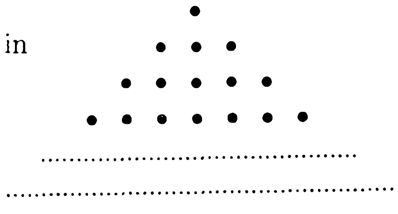
(A) (25, 47) (B) (25, 49) (C) (25, 37) (D) (25, 97)

Solution: The first member of the 25^{th} term in the square (1,2), (2,3), (3,5), (4,7), (5,11), (6,13) is clearly 25. The second member of each term is a prime number, the second members of $1^{\text{st}}, 2^{\text{nd}}, 3^{\text{rd}}, \cdots 25^{\text{th}}$ terms are the $1^{\text{st}}, 2^{\text{nd}}, 3^{\text{rd}}, \cdots 25^{\text{th}}$ prime numbers.

The 25^{th} prime number in the prime number sequence $2, 3, 5, 7, 11, 13, \cdots$ (47 and 37 are the 15^{th} and 12^{th} prime numbers respectively). Answer: (D)

29. The total number of dots in the first 100 rows is

(A) 550 (B) 560
(C) 5500 (D) 10000



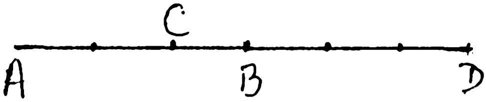
Solution: It is clear that the number of dots in the n^{th} row is the n^{th} odd number.

So total number of dots in the first 100 rows is $1 + 3 + 5 + 7 + \cdots + 199$ (observe that the 100^{th} odd number is $2 \times 100 - 1$). The total number of the 1^{st} 100 odd numbers which is $100^2 = 10000$ (Refer solution of 1st problem screening test). Answer: (D)

30. There are four points A, B, C, D on a straight line. The distance between A and B is 3cms. C and D are both twice as far from A as from B. Then the distance between C and D is

(A) 1cm (B) 2cms (C) 3cms (D) 4cms

Solution:



Given $AB = 3$ cm and $\frac{AC}{CB} = \frac{2}{1}$; $AB = AC + CB = 2CB + CB = 3CB = 3$. $\therefore CB = 1$ i.e., C divides AB internally in the ratio $2 : 1$. If D must also satisfy a similar condition then D should either coincide with C or it is the point which divides AB externally in the ratio $2 : 1$.

Given $\frac{AD}{DB} = 2$. If $BD = x, AD = AB + BD = 3 + x$.
So $\frac{3+x}{x} = 2$ or $3 + x = 2x$ or $x = 3$.
$\therefore CD = CB + BD = 1 + 3 = 4$ cm. Answer: (D)

31. Which is the odd one out?

(A) $1 \div 9 + 9 \div 1$ (B) $1 \times 9 \div 9 \times 1$
(C) $1 - 9 + 9 \times 1$ (D) $1 + 9 \div 9 - 1$

Solution: The values of the expression in (B), (C) and (D) are all equal to 1; but the value of expression in (A) is $9\frac{1}{9}$. Hence the odd one out is (A). Answer: (A)

Aliter: If we look at the problem from a different angle, of the four algebraic operations $+, -, \times$ and $\div$ option (C) is the one with no $\div$ involved. Hence it can also be termed the odd one out. Answer: (C)

32. I set my video recorder to record a late night programme from 11-15 p.m. to 1-05 a.m. How many minutes of the new 4 hour tape remained unused?

(A) 50 (B) 110 (C) 130 (D) 150

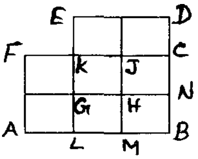
Solution: Time from 11-15 p.m. to 12-00 mid-night is 45 minutes and time from 12-00 midnight to 1-05 a.m. is 65 minutes.

Hence recorded duration = 110 minutes.

The remaining time available in the tape for recording is
4 × 60 − 110 or 130 minutes. Answer: (C)

33. How many squares are there altogether in this diagram?

(A) 8 (B) 9 (C) 10 (D) 11

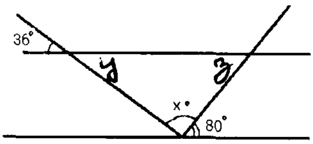


Solution: There are eight 1 × 1 squares, there are three 2 × 2 squares (AMJF, LBCK, GNDE). There is no, 3 × 3 square.

∴ Total number of squares = 8 + 3 = 11. Answer: (D)

34. How big is the angle x? if l_1 is parallel to l_2?

(A) 30° (B) 36°
(C) 44° (D) 64°



Solution: The two lines l_1 and l_2 are parallel lines. Then using the angular properties of a pair of parallel lines cut by a transversal we have

$$y = 36^\circ, z = 80^\circ \text{ and } x + y + z = 180^\circ$$

So $x = 64^\circ$. Answer: (D)

35. How many of these statements are true?

(1) $12 \div \frac{1}{2} = 6$ (2) $3\% = 0.3$ (3) $\frac{1}{7} < \frac{1}{9}$ (4) $0.2 \times 0.4 = 0.8$

(A) None (B) One (C) Two (D) Three

Solution: Obviously, none of the statements is true.

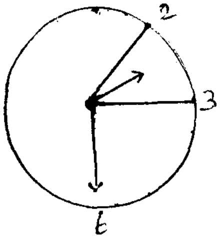
Answer:(A)

36. What is the angle between the two hands of a clock at 2-30?

(A) 100° (B) 105° (C) 120° (D) 110°

Solution: At 2-30 the hour hand is precisely midway between 2 and 3 and the minute hand is on 6. Hence the angle between the two hands is $90^\circ + \frac{1}{2} \times 30^\circ = 105^\circ$.

Answer: (B)



37. A quiz has 20 questions with seven points awarded for each correct answer; two points deducted for each wrong answer and zero mark given for each question omitted. Ram scores 87 points. How many questions did he omit?

(A) 2 (B) 5 (C) 7 (D) 9

Solution: Since 7 points are awarded for each correct answer, and the total score of Ram is 87, Ram should have done more than 12 sums correct. Supposing he has done 13 sums correct, he would have scored $13 \times 7 = 91$ marks. But his score is 87, 4 less than 91. These 4 marks are deducted for doing 2 sums wrong. Thus Ram would have done 15 sums of which 2 sums are wrong and did not attempt 5 sums. (check if Ram has done 14 which is more than 12) correct he would have scored 98 from which $(98 - 87) = 11$ marks are deducted for doing $\frac{11}{2}$ sums wrong. As $\frac{11}{2}$ is not an integer, it is not possible. If he had done 15 sums correct then he would has score 105 marks from which $(105 - 87) = 18$ marks are deducted for doing 9 sums wrong, this makes the total sums attempted as $15+9 = 24$ and this is not possible as there are only 20 sums given. Answer: (B)

Aliter: From (1) and (2) above, we can also get

$$(7x - 2y) - 7(x + y + z) = 87 - 7 \times 20$$

where x is correct answered, y wrong answered and z not answered queries.

or $9y+7z = 53$. When $y = 0$ (or 1), $7z = 53$ (or 44) and z cannot be an integer. But when $y = 2$, we have $7z = 53 - 18 = 35$ or $z = 5$. Of course, then $x = 13$. Number of questions omitted is 5. Answer: (B)

38. Nine bus stops are equally spaced along a bus route. The distance from the first to the third is 600m. How far is it from the first stop to the last?

(A) 800m (B) 1600m (C) 1800m (D) 2400m

Solution: Nine bus stops equally spaced create 8 equal intervals. From first to third there are 2 intervals = $600m$. So, each interval is 300m. Hence the required distance is $8 \times 300 = 2400m$. Answer: (D)

39. Sound travels at about 330 meters per second; light travels so fast that it arrives almost instantaneously. If the gap between a flash of lightning and its clap of thunder is 6 seconds, roughly how far away is the cloud?

(A) 55m (B) 330m (C) 2kms (D) 1km

Solution: Time gap =6sec; speed of sound =330m / sec.

$\therefore$ Distance of the storm is $6 \times 330 = 1980m$ =2km, roughly.

Answer: (C)

40. Each letter stands for a different digit. Which letter has the lowest value?

$$\begin{array}{r} A\ M\ \ + \\ 4\ \ \ \\ \hline T\ I\ C\ \ \\ \hline \end{array}$$

(A) A (B) M (C) T (D) I

Solution: A two digited numbers AM added to 4 will give the three digited number TIC only if $A = 9$ and $T = 1$ and the carry over of $M + 4$ is 1. M has to be at least 6 for $M + 4$ to be two digited when $M = 6, C = 0$. In this case C will have the lowest value. But then I and C will both the zero.

To avoid this we can take M to be 7 so that $C = 1$ then also $I = 0$. But now T and C have the same value 1.

To overcome this we can take $M = 8$ so that $C = 2$, $A = 9, I = 0$ and $T = 1$. Hence I has the lowest value 0 with A, M, T, I and C all different.

Answer: (D)

41. The number of 3 digit numbers which end in 7 and are divisible by 11 is

(A) 2 (B) 4 (C) 6 (D) 8

Solution: Let the digits of the number be a, b and 7 in that order. Then the number is divisible by 11 if $a + 7 = b$, or the difference between $a + 7$ and b is a multiple of 11. Note that $a + 7$ lies in the range from 8 to 16, and b lies in the range from 0 to 9. Hence, either $a + 7 = b$ or $a + 7 = b + 11$.

Case 1: $a + 7 = b$.

We get the numbers 187 and 297.

Case 2: $a + 7 = b + 11$; that is, $a = b + 4$.

We have the numbers 407, 517, 627, 737, 847 and 957.

Thus, there are $2 + 6 = 8$ three digit numbers with last digit 7 and divisible by 11. Answer (D)

42. In a six digit number 5 digits are prime numbers. The sum of all the digits is 24. The 2nd, 3rd and 5th digits are identical and the others are distinct digits. The number is divisible by 4. The last digit of the number is

(A) 2 or 4 (B) 4 or 6 (C) 4 or 6 or 8 (D) 2 or 6 or 8

Solution: Since 5 of the six digits are prime numbers and second, third and fifth digits are identical, these digits should be prime numbers. The number is divisible by 4 and therefore the end digit should be 0,2,4,6 or 8. Fixing end digit as 0, the fifth digit and hence second and third should be only 2. Sum of these 4 digits is $2 + 2 + 2 + 0 = 6$. The other two digits should be prime and distinct. Taking the biggest possible values for the other digits as 7 and 5, the sum of all the 6 digits becomes $7+5+2+2+2+0=18\neq 24$. So the end digit cannot be 0.

Fixing end digit as 2, second, third and fifth as 3, we get the sum of these four digits as $9+2=11$. So the other two digits together should give 13. One of them is a prime number. We can have 8 and 5 to be the other digits. Thus 833532 is a number which is divisible by 4, and there are 5 prime digits and the sum of the digits is 24. So, 2 can be an end digit.

Now, fixing 4 as the end digit, the 5th digit being a prime number, it could only be 2 (why?). In this case, the sum of their four digits is $3\times 2+4=10$, the other two prime digits should add up to 14, which is impossible; so 4 cannot be the end digit.

By similar arguments, we can see that the numbers can have 6 or 8 as the end digit. Examples: 733236 and 322728. Thus the end digits of the six digit numbers with the given conditions can be 2 or 6 or 8. Answer: (D)

43. In an isosceles acute angled triangle one angle is 50°.

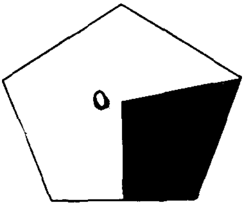
I. The other two angles are 65° and 65°.
II. The other two angles are 50° and 80°
Then which one of the above statements can be true?

(A) I only (B) II only (C) I and II both
(D) either I or II but not both

Solution: One angle is 50°; the other two angles 65° each, give 65, 65, 50° and (the sum of angles is 180°, so these triangles exist and are isosceles. One angle is 50°, the other angles are 50° and 80°, gives the angles of the triangle to be 50°, 50°, 80° these type of $\triangle$'s exist and are isosceles. So, both the statements can be true.

Answer (D)

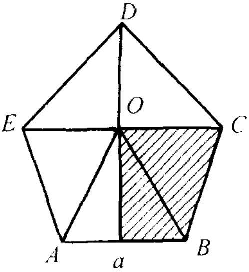
44. O is the centre of the regular pentagon. What part of the whole pentagon is the shaded region?



(A) 10% (B) 20% (C) 25%
(D) 30%

Solution: When the centre O is joined to the vertices A, B, C, D and E of the regular pentagon, we get, 5 (congruent) triangles of equal area.

The shaded portion is one full triangle + half the ΔAOB. Thus the ratio of the shaded region to the whole pentagon is $1\frac{1}{2} : 5$ or 3:10, which gives the shaded portion to be $\frac{3}{10} = 30\%$, of the whole region. Answer: (D)



45. An isosceles triangle has equal sides 7 units long and the length of the third side is an integer. The number of such triangles is

(A) 13 (B) 11 (C) 9 (D) 7

Solution: In any triangle, the sum of the lengths of any two sides is greater than the third side. Therefore, in this case the third side with integer length is less than 14 units. So the third side can be from 1 unit to 13 units and there are 13 such triangles. Answer: (A)

46. How many positive integers less than 100 can be written as the sum of 9 consecutive positive integers?

(A) 11 (B) 9 (C) 7 (D) 5

Solution: The nine consecutive integers can be from

(1) 1 to 9 (2) 2 to 10 (3) 3 to 11 (4) 4 to 12 (5) 5 to 13 (6) 6 to 14 (7) 7 to 15.(8 to 16 gives the sum 108.)

The sums are 45, 54, 63, 72, 81, 90, 99.Thus there are 7numbers less than 100 which can be expressed as a sum of 9consecutive integers. Answer: (C)

47. In a month three Tuesdays were on even dates. Then, 21st of the month is a

(A) Sunday (B) Monday (C) Wednesday (D) Saturday

Solution: Look at the dates in a calendar on which the same day of the week occurs in a month. They can be 1, 8, 15, 22, 29; or 2, 9, 16, 23, 30; etc. Three even dates occur only in the second case. So Tuesday occurs on 2, 9, 16, 23 and 30. The 21st is Sunday. Answer: (A)

48. What is the first digit of the smallest number whose sum of the digits is 2007?

(A) 9 (B) 8 (C) 3 (D) 2

Solution: The smallest number results when all the digits or (most possible digits) are 9. Thus $\frac{2007}{9} = 223$ digit number, with each digit being 9, is the smallest number with digital sum 2007. So the first digit of the number is also 9.

Answer: (A)

49. The product of two integers is $2^7 \cdot 3^3 \cdot 5^5 \cdot 7^3$. Then the sum of the two numbers may be divisible by

(A) 16 (B) 9 (C) 25 (D) 49

Solution: If a number d divides a number a, but does not divide another number b, then d does not divide $a+b$ (or $a-b$).

Here, we are given that the product of the two numbers, say A and B, is equal to $2^7 \cdot 3^3 \cdot 5^5 \cdot 7^3$. If $A = 2^m M$ and $B = 2^n N$, where M and N are not divisible by 2, then $m + n = 7$. So either $m \geq 4$or $n \geq 4$, but not both. Thus, we see that either A or B, but not both, is divisible by $2^4 = 16$.

Similarly, we find that either A and B, but not both is divisible by $3^2 = 9$. The same is true with 7^2 instead of 3^2. However, both A and B can be divisible by $5^2 = 25$. For example, $A = 5^2$ and $B = 2^7 \cdot 3^3 \cdot 5^5 \cdot 7^3$. Answer: (C)

50. Let a, b, c, d be positive integers where $a + b + c = 53$, $b+c+d = 51$, $c+d+a = 57$ and $d+a+b = 58$. Then the greatest and the smallest numbers among a, b, c, d are respectively

(A) b and d (B) a and c (C) c and a (D) d and b

Solution:

$$a + b + c = 53 \quad (1)$$

$$b + c + d = 51 \quad (2)$$

$$c + d + a = 57 \quad (3)$$

$$d + a + b = 58. \quad (4)$$

Adding (1)(2)(3)and (4), $3(a + b + c + d) = 219$

$a + b + c + d = 73$

$c = (a + b + c + d) - (d + a + b) = 73 - 58 = 15$

$a = (a + b + c + d) - (b + c + d) = 73 - 51 = 22$ and

$b = 73 - 57 = 16$ and $d = 73 - 53 = 20$.

Clearly a is the biggest number and c is smallest number.

Answer: (B)

51. The weight of a dog is 12 kg more than one fourth of its weight. Then the weight of the dog is

(A) 14 kg (B) 15kg (C) 16 kg (D) 18 kg

Solution: If the weight the dog is $d$kg, one fourth the weight of the dog is $\frac{1}{4}d$ kg, and hence

$12 + \frac{1}{4}d = d \Longrightarrow \frac{3}{4}d = 12 \Longrightarrow d = \frac{12\times 4}{3} = 16$ kg

i.e., weight of the dog is 16 kg. Answer: (C)

52. Define $a * b = 2a + 2b - ab$, a, b are any two numbers.

For example, $4 * 3 = 2 \times 4 + 2 \times 3 - 4 \times 3 = 8 + 6 - 12 = 2$.

If $3 * x = 4$, then x is

(A) 2 (B) 4 (C) 6 (D) 8

Solution:

$$\begin{aligned} 3 * x = 4 &\Rightarrow 2 \times 3 + 2 \times x - 3 \times x = 4 \\ &\Rightarrow 6 + 2x - 3x = 4 \\ &\Rightarrow 6 - x = 4 \Rightarrow x = 2 \end{aligned}$$

Answer: (A)

53. The number of prime numbers less than 100 whose sum of the digits is 2 is

(A) 1 (B) 2 (C) 3 (D) 4

Solution: The number less than 100, with digital sum 2 is 20,11,02 or 2 as 2 can be expressed as $0 + 2$, $1 + 1$ or $2+0$.

Of these numbers, 11 and 2 are the prime numbers.

Answer: (B)

54. If all the divisors of 128 are arranged in ascending order the sixth divisor is

(A) 16 (B) 64 (C) 32 (D) 128

Solution: $128 = 2^7$ the powers of two, less than 2^8 are $1 = 2^0, 2 = 2^1, , 2^2, 2^3, 2^4, 2^5, 2^6$ and 2^7. Only these are divisors of $128 = 2^7$, and the sixth of the divisors is $2^5 = 32$. Answer: (C)

55. Consider the fractions $\frac{1}{10}, \frac{2}{9}, \frac{3}{8}, \cdots, \frac{9}{2}, \frac{10}{1}$. The number of these fractions which are irreducible is

(A) 10 (B) 8 (C) 6 (D) 4

Solution: If the fraction are reducible, then the numerator and denominator have the greatest common divisor greater than 1.

Example: $\frac{12}{30} = \frac{2\times6}{5\times6} = \frac{2}{5} \times \frac{6}{6} = \frac{2}{5} \times 1 = \frac{2}{5}$. Thus the numerator 12 and denominator 30 have 6 as the greatest common divisor, and its irreducible form is $\frac{2}{5}$.

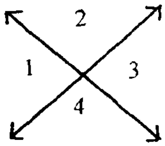
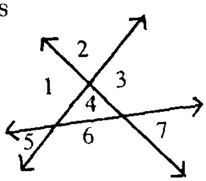
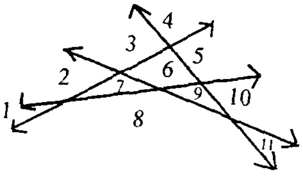
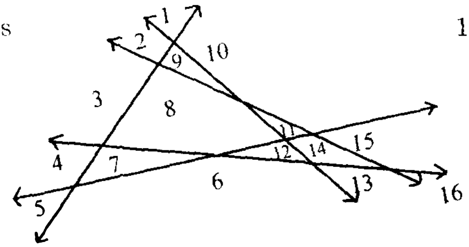
Now 6 divides 12 and 6 divides 30 and 6 divides $12 + 30 = 42$ also. Thus if the numerator and denominator has the gcd 'd', then d divides the sum of the numerator and denominator.

In the above problem the sum of the numerator and denominator of each of the 10 fractions is 11, which is a prime number, with divisors 1 and 11. So if any number greater than 1 divides numerator and denominator then it should divide 11 also. Since there exists no such number the numerator and denominator are coprime numbers and hence all the 10 fractions are irreducible. Answer: (A)

56. If five lines are drawn in the plane the maximum number of regions into which the plane is divided is

(A) 10 (B) 12 (C) 14 (D) 16

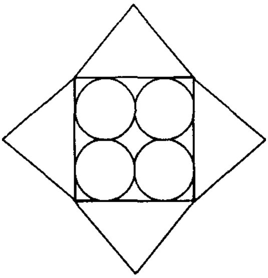
Solution: Draw some lines and observe the patterns

Number of lines	Number of regions	Increase in the number of region
One line 	2	1
Two lines  	4	2
Three lines  	7	3.
Four lines  	11	4
Five lines  1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	16	5

Answer: (D)

Note: In this problem, to get the maximum number of regions we should have the conditions that (1) no three lines are concurrent (2) no two lines are parallel. From the last column, where we get the increase of regions in successive stages, we see that if there are 6 lines, there should be $16 + 6 = 22$ regions. If there are 7 lines there should be $22 + 7 = 29$ region. You can use the formula $R = \frac{1}{2}l^2 + \frac{1}{2}l + 1$ where R is the number of regions and l is the number of lines to get the maximum number of regions.

57. Bases of four equilateral triangles form a square. Inside the square four circles of radius 5 units are drawn as in the figure. The perimeter of the four cornered star is



(A) 100 (B) 120

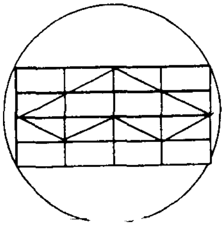
(C) 160 (D) 200

Solution: Since the radius of the circle is 5 units, it is clear from the figure, the side of the square is $4 \times 5 = 20$ units and since the 4 cornered star is made up of 4 equilateral triangles where base is 20 units, length the perimeter,of the star is

$$8 \times 20 \text{ units} = 160 \text{ units}$$

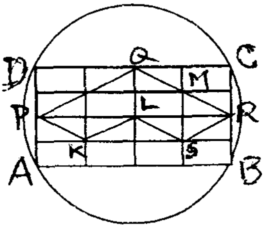
Answer: (C)

58. The diameter of the circle in the picture is 10 cms. The perimeter of the region marked in thick line in cms. is



(A) 12 (B) 16 C) 20(D) 24

Solution: The diameter of the circle in the picture is the length $AC = 10\text{cm} = AK + KL + LM + MC$. But all the 16 small rectangles are identical. Thus the diagonals

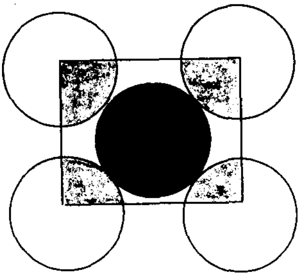


$AK = KL = LM = MC = \frac{10}{4} = 2.5\text{cm}$. So the perimeter and $PK = KL = LS = SR = 2.5\text{cm}$ of the — lined region is

$$= PQ + QR + RS + SL + LK + KP$$
$$= 5 + 5 + 2.5 + 2.5 + 2.5 + 2.5 = 20\text{cm}$$

Answer: (C)

59. Four circles of equal radii are centred at the four vertices of a square. These 4 circles touch a fifth circle of equal radius placed inside the square. The ratio of the shaded area of the circles to the unshaded area of the circles is



(A) 1:3 (B) 2:3 (C) 3:4 (D) 2:5

Solution: The shaded area of the 5 circles is equal to area of one full circle plus areas of 4 quarter circles; that is, equal to area of 2 circles of the same radius. The area of the unshaded portion of the four circles at the corner is $4 \times \frac{3}{4}$ of the circles = area of 3 such circles. So the required ratio is 2:3. Answer: (B)

60. Mala, Devi, Sita, Emma and Kala are sitting in a park. Mala is not sitting on the farthest right and Devi is not sitting on the farthest left. Sita is not sitting at either the farthest left or farthest right. Kala is not sitting next to Sita and Sita is not sitting next to Devi. Emma is sitting to the right of Devi but not necessarily next to her. Which girl is sitting farthest to the right?

(A) Devi (B) Sita (C) Emma (D) Kala

Solution: Since Emma is sitting right of Devi, Devi is not sitting at the farther right nor at the farthest left.

Sita also is neither sitting at the farther right nor at tarthest left; Mala is not sitting on the farthest right.

Thus Kala or Emma must be sitting on the farther right.

If the positions are numbered from left to right as 1,2,3,4,5, then the 5^{th} place should be occupied by Kala or Emma. Suppose that Kala is in the 5^{th} place.

Then 2,3 and 4 should be occupied, by Devi,Sita and either Emma or Mala. Since Sita is not sitting next to Devi and Kala not sitting next to Sita, Sita can occupy only 2^{nd} place and Devi is 4^{th} and Emma is $3rd$ or $1st$.

This is not possible since Emma is sitting to the right of Devi. Hence Kala cannot be in the $5th$ place. Answer: (C)

Note: Kala, Devi, Mala, Sita, Emma is the order.

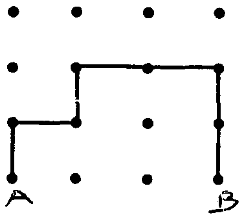
61. The natural numbers are written as below following some rule $1, 3, 6, 10, \cdots$. The tenth number is
(A) 55 (B) 62 (C) 105 (D) 35

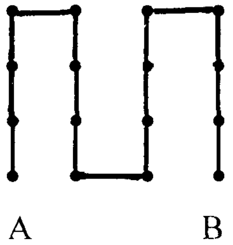
Solution: The numbers are $1, 3, 6, 10, \cdots$. The successive differences are 2,3,4. Thus the difference between 10 and the next term must be 5. The next number is 15. Extending this pattern we get $1, 3, 6, 10, 15, 21, 28, 36, 45, 55$.
$\therefore$ Tenth number is 55. Answer:(A)

62. In the adjoining figure we have 16 dots equally spaced in 4×4 grid. From A to B one has to go. He can go up or down, left or right from one dot to the other. In the figure the length of the path is 7. The maximum length of the path is

(A) 12 (B) 13 (C) 15 (D) 16

Solution: All the points are visited once. The maximum length of the path is 15 Answer: (C)





63. $20072007 \div 10001$ is a

(A) Three digit number (B) Four digit number
(C) Eight digit number (D) Six digit number

Solution: $10001 \times 2007 = 20072007$.

From this we get $\frac{20072007}{10001} = 2007$.
$\therefore$ It is a four digit number. Answer: (B)

64. It is given that $5\frac{3}{a} \times b\frac{1}{2} = 19$(where the two fractions are mixed fractions); then $a + b$
(A) 10 (B) 12 (C) 9 (D) 15

Solution: $5\frac{3}{a} \times b\frac{1}{2} = 19$.

Clearly the possible values of b are less than 4.

If $b = 1$ or 2 we get the product less than 19.

Let $b = 3$, $5\frac{3}{a} \times 3\frac{1}{2} = 19$. (i.e.,) $(5 + \frac{3}{a}) \times \frac{7}{2} = 19$

$\Rightarrow 5 \times \frac{7}{2} + \frac{3}{a} \times \frac{7}{2} = 19$

$\Rightarrow \frac{3}{a} = \frac{3}{7} \Rightarrow a = 7 \;\therefore\; a + b = 10.$ Answer: (A)

Another Method:

$$\begin{aligned} 5\frac{3}{a} = \frac{5 \times a + 3}{a} &= \frac{5a+3}{a} \\ b\frac{1}{2} &= \frac{2b+1}{2} \\ 5\frac{3}{a} \times b\frac{1}{2} = \frac{(5a+3)}{a} \times \frac{2b+1}{2} &= 19 \\ (5a+3)(2b+1) &= 38a \end{aligned}$$

Clearly in $5\frac{3}{a}$, a should be greater than 3.

$a = 4$ gives $23(2b+1) = 38 \times 4$ and 23 does not divide 38×4. Why? $\therefore a \neq 4$.

$a = 5$ gives $28(2b+1) = 38 \times 5$
$(2b+1) = \frac{190}{28} = \frac{95}{4} = 6\frac{11}{14}$ and $(2b+1)$ is an integer.

$\therefore a \neq 5$

$a = 6 \Rightarrow 33(2b+1) = 38 \times 6$. Again, this equality does not give integer value to 6.

$a = 7 \Rightarrow 38(2b+1) = 38 \times 7 \quad \Rightarrow 2b + 1 = 7$
$b = 3$. $\therefore a + b = 7 + 3 = 10$.

65. a and b are two primes of the form p and $p+1$, and $M = a^a + b^b$; $N = a^b + b^a$ then

(A) M and N are composite
(B) M is a prime but N is composite
(C) M and N are primes
(D) M is composite, N is prime

Solution: The only consecutive primes are 2 and 3

$$\begin{aligned} M &= 2^2 + 3^3 = 4 + 27 = 31 \\ N &= 2^3 + 3^2 = 8 + 9 = 17 \end{aligned}$$

We find that M, N are primes. Answer: (C)

66. A's salary is 50 % more than B's. How much percent is B's below A's?

(A) 50% (B) 25% (C) 75% (D) $33\frac{1}{3}\%$

Solution: Let B's salary be 100.
Given: A's salary is 50% more than B's.
A's Salary = Rs. $100 \times \frac{150}{100} = 150$. B's % below A's is

$$\begin{aligned} &= \frac{\text{difference of their salaries}}{A\text{'s salary}} \times 100 \\ &= \frac{150 - 100}{150} \times 100 \\ &= \frac{50}{150} \times 100 = 33\frac{1}{3}\% \end{aligned}$$

Answer: (D)

67. $S = 1 + 22 + 333 + 4444 + 55555 + 666666 + 7777777$.

The digits of the number S are added. If you get a double digit number, again add the digits and continue to get a single digit number. The final single digit is

(A) 9 (B) 5 (C) 8 (D) 1

Solution: Method 1: If the unit digits are added we get 28.

$$
\begin{array}{ccccccc}
 & & & & & & 1 \\
 & & & & & 2 & 2 \\
 & & & & 3 & 3 & 3 \\
 & & & 4 & 4 & 4 & 4 \\
 & & 5 & 5 & 5 & 5 & 5 \\
 & 6 & 6 & 6 & 6 & 6 & 6 \\
7 & 7 & 7 & 7 & 7 & 7 & 7 \\
\hline
8 & 5 & 0 & 4 & 7 & 9 & 8 \\
\hline
\end{array}
$$

If the unit digits are added we get 28.

The second column sum is one less than 28 (i.e.) 27.
Third column sum is two less than 27 (i.e.) 25 etc.
$8+5+0+4+7+9+8=41$ $4+1=5$ Answer: (B)

Method 2: The sum of the digits of the sum is equal to the sum of the digits of the individual numbers

sum of the digits of the numbers

$$=1+4+9+16+25+36+49=140$$

Sum of the digits $= 1+4+0=5$.

Method 3: Try finding such sum of the digits deleting the number 9.

We have	$1+4+9+16+25+36+49$
Digit sum	$1+4+9+7+7+9+13$
-do- with out 9's	$1+4+7+7+\cancel{9}+4$
-do-	$5+7+7+4$
-do-	$12+11$
-do-	$\cancel{9}+3+\cancel{9}+2$
-do-	$3+2=5$

68. The value of $\frac{1}{2+\frac{1}{3+\frac{1}{4+\frac{1}{5}}}}$.

(A) $\frac{77}{60}$ (B) $\frac{68}{157}$ (C) $\frac{2007}{2008}$ (D) none of these

Solution: Given fraction $= \dfrac{1}{2+\frac{1}{3+\frac{1}{\left(\frac{21}{5}\right)}}}$

$$= \frac{1}{2+\frac{1}{3+\left(\frac{5}{21}\right)}} = \frac{1}{2+\frac{1}{\left(\frac{68}{21}\right)}}$$

$$= \frac{1}{2+\frac{21}{68}} = \frac{1}{\left(\frac{157}{68}\right)} = \frac{68}{157}.$$

Answer: (B)

69. Which one of the following is incorrect

(A) Doubling the length of a rectangle doubles the area
(B) Doubling the altitude of a triangle doubles the area
(C) Doubling a given quantity may make its area lesser than the original
(D) Doubling the radius of a circle doubles the area.

Solution: Area of a circle is πr^2 where r is the radius.
If r is doubled the area is $\pi(2r)^2 = 4\pi r^2$.
D is wrong. Answer: (D)

Note: (C) is also wrong. Both (D) and (C) are correct answers.

70. If $\dfrac{4^7+4^7+4^7+4^7}{2^7+2^7} = 2^x$, then $x =$

(A) 7 (B) 8 (C) 9 (D) 2^7

Solution:

$$\frac{4^7+4^7+4^7+4^7}{2^7+2^7}=\frac{4\times 4^7}{2\times 2^7}=\frac{4^8}{2^8}=\frac{2^{16}}{2^8}=2^8$$

$\therefore\ 2^8 = 2^x;\ \boldsymbol{x = 8}.$ Answer: (B)

71. The first term of a sequence is $\frac{3}{8}$. If a is a term the next term is $\frac{1-a}{1+a}$. Then the 2007th term is

(A) $\frac{5}{11}$ (B) $\frac{8}{3}$ (C) $\frac{3}{8}$ (D) 1

Solution: First term$=\frac{3}{8}$

$$\begin{aligned}\text{Second term} &= \frac{1-\frac{3}{8}}{1+\frac{3}{8}}=\frac{5}{11}\\ \text{Third term} &= \frac{1-\frac{5}{11}}{1+\frac{5}{11}}=\frac{6}{16}=\frac{3}{8}\\ \text{Fourth term} &= \frac{1-\frac{3}{8}}{1-\frac{3}{8}}=\frac{5}{11}\end{aligned}$$

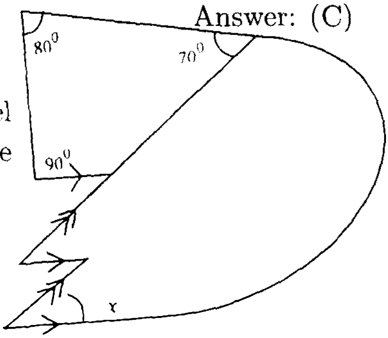
we have $\frac{3}{8},\frac{5}{11},\frac{3}{8},\frac{5}{11},\cdots$.

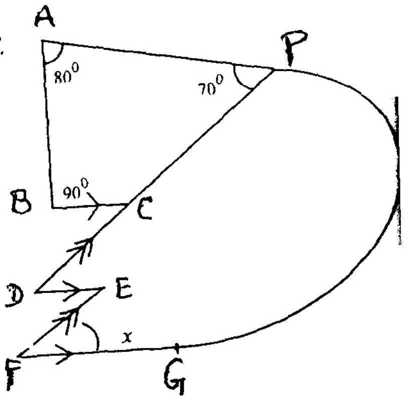
All the odd terms are $\frac{3}{8}$. 2007^{th} term is $\frac{3}{8}$.

Answer: (C)

72. In the adjoining figure the parallel lines are marked by arrow lines. The value of the angle x is

(A) 60° (B) 70°
(C) 80° (D) 50°





Solution: In quadrilateral $ABCP$.

$$\angle BCP + 90^\circ + 80^\circ + 70^\circ = 360^\circ$$

$$\begin{aligned} \Rightarrow \widehat{BCP} &= 120^\circ \\ \Rightarrow \widehat{BCD} &= 60^\circ \\ \widehat{CDE} &= 60^\circ \text{alternate angles} \\ &= \angle DEF = \angle EFG \end{aligned}$$

$x = \angle DEF$ alternate angle between two pairs of parallel lines $\Rightarrow x = 60^\circ$. Answer: (A)

73. The value of $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \frac{1}{4\times5} + \frac{1}{5\times6} + \frac{1}{6\times7} + \frac{1}{7\times8}$ is

(A) $\frac{8}{7}$ (B) $\frac{7}{8}$ (C) 1 (D) $\frac{1}{1569}$

Solution:

$$\begin{aligned} \frac{1}{1\times2} &= \frac{1}{1} - \frac{1}{2} \\ \frac{1}{2\times3} &= \frac{1}{2} - \frac{1}{3} \\ \vdots \quad & \vdots \quad \vdots \\ \frac{1}{7\times8} &= \frac{1}{7} - \frac{1}{8} \end{aligned}$$

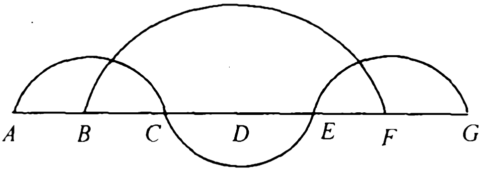
Add all $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \frac{1}{4\times5} + \frac{1}{4\times5} + \frac{1}{5\times6} + \frac{1}{6\times7} + \frac{1}{7\times8}$

$$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{7} - \frac{1}{8}$$

$$= 1 - \frac{1}{8} = \frac{7}{8}.$$

Answer: (B)

74. In the adjoining figure AG=6cm.



$AB = BC = CD = DE = EF = FG =$ 1cm.
Semicircles are drawn as in the diagram.

The total length of the path A to G along the smaller semicircles is x. The length of the path (B to F) along the bigger semicircle is y. Then

(A) $x < y$ (B) $x = y$ (C) $x > y$ (D) $x + y = 6\pi$

Solution:

Given $AB = BC = CD = DE = CF = FG =$ 1cm.

Length of the path form A to G along smaller semicircles

$= x = \pi(1 + 1 + 1) = 3\pi$cm

Length of the path form B to F along the bigger semicircle

$= y = \pi \cdot 2 = 2\pi$. Clearly $x > y$. Answer: (C)

75. Three cubes of metal whose edges are 3cm, 4cm and 5cm are melted and formed into a single cube. If there is no wastage in the process, the edge of the new cube is

(A) 12cm (B) 7cm (C) 9cm (D) 6cm

Solution: Edges of the three cubes are 3cm, 4cm, 5cm. Sum of the volumes of the cubes $= 3^3 + 4^3 + 5^3 = 216 = 6^3$.

$\therefore$ Edge of the new cube is 6cm Answer: (D)

76. a, b, c are any three of the first four prime numbers. $n = a^2bc$. The biggest and the smallest value of n are respectively

(A) (1035, 50) (B) (735, 60) (C) (525, 50) (D) None of these

Solution: The first four prime numbers are $2, 3, 5, 7$

Biggest value of $n = a^2bc$ is $7^2 \times 5 \times 3 = 735$

Smallest value of n is $2^2 \times 3 \times 5 = 60$. Answer: (B)

77. The number of rectangles with integer sides and with perimeter 16 cm is

(A) 8 (B) 4 (C) 3 (D) 1

Solution: Let l, b be the length and breadth of a rectangle whose perimeter is 16

$$\therefore\ 2(l + b) = 16$$

$$\Rightarrow\ l + b = 8$$

l	b
1	7
2	6
3	5
4	4

In this $l = b = 4$ forms a square. But all squares are rectangles.

$\therefore$Number of rectangles $= 4$. Answer: (B)

Note: All rectangle are not squares!

78. A number when divided by 899 gives a remainder 63. What remainder will be obtained by dividing the same number by 29?

(A) 5 (B) 18 (C) 19 (D) 21

Solution: Let x be the number when divided by 899 gives a remainder 63. Let its quotient be y.

$$\begin{aligned} \therefore\ x &= 899 \times y + 63 \\ x &= 29 \times 31 \times y + 29 \times 2 + 5 \\ &= 29[31y + 2] + 5 \end{aligned}$$

$\therefore$ When x is divided by 29 we find 5 is the remainder.

Answer: (A)

79. In 1 KM race when A wins B by 40 metres. A wins C by 70 metres. Then when B and C run

(A) B wins C by 110m (B) C wins B by 100m
(C) B wins C by $31\frac{1}{4}$m (D) C wins B by 50m

Solution: Let 1000m be the total distance.

Note: Question should have included that the distance to be covered for the race is 1 KM. Otherwise one may not be able to get the solution given.

When A covers 1000 B covers $(1000 - 40) = 960$m

When A covers 1000 C covers $(1000 - 70) = 930$m

When B runs 960m C runs 930m

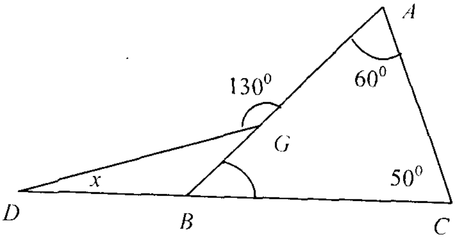
$\therefore$ B runs 1000m when C runs $\frac{1000 \times 930}{960}\text{m} = 968\frac{3}{4}\text{m}$

B wins C by $(1000 - 968\frac{3}{4}) = 31\frac{1}{4}\text{m}$

Answer: (C)

80. In the adjoining figure
$x =$

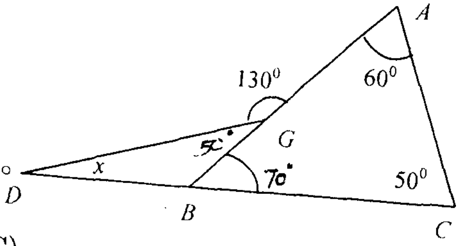
(A) 40° (B) 30°
(C) 20° (D) 10°



Solution:

$\widehat{ABC} = 70°$, $\widehat{DGB} = 50°$

$\widehat{GDB} = \widehat{GBC} - \widehat{DGB} = 20°$



Answer: (C)

FINAL TEST

1. Find all the three digit numbers formed by 3,5 and 7 in which no digit is repeated. For example if you do the same for 1,2,3 we have 123,231,213 as some of the numbers that you can get. Add all of them and divide the sum by 3 + 5 + 7. Call the number that you get as a. Now find all the three digit numbers that are formed by 2,6 and 8, again without repetitions. Add all of them and divide the sum by 2 + 6 + 8. Call the number you get as b. Compare a and b to find which is bigger.

   Solution: If we add the six 3-digit numbers that we get using the three digits 3,5,7 without repeating any digit we get

   $$357 + 375 + 537 + 573 + 735 + 753 = 3330.$$

   The sum of the digits is $3 + 5 + 7 = 15$. Hence we get

   $$a = \frac{3330}{15} = 222.$$

   If we do the same with the digits 2, 6, 8 we get

   $$268 + 286 + 628 + 682 + 826 + 862 = 3552.$$

   The sum of the digits is $2 + 6 + 8 = 16$. Hence

   $$b = \frac{3552}{16} = 222.$$

   So the answer to our question is $a = b$.

   Note: This is no surprise! If we do the same with three digits a, b, c, we will get

   $$2(a + b + c)(1 + 10 + 100) = 2(a + b + c)(111)$$

   as their sum. When we divide this sum by $a + b + c$ we will always get $2(111) = 222$.

2. Ram bought a notebook containing 98 sheets, and numbered them from 1 to 196. Krishna tore 35 sheets of Ram's notebook and added the 70 numbers he found on the pages. Could Krishna have got 2004 as the sum?

   Solution: No is the answer. The sum of the page numbers that appear on the two sides of each page is odd (why?) as the sum of any two consecutive numbers is always odd. So when we add 35 odd numbers it is always odd and it cannot be 2004.

3. There are 20 cities in a certain country. Every pair of cities is connected by an air route. How many air routes are there?

   Solution: The number of air routes connecting the first city C_1 with the remaining 19 cities is 19. Again the number of air routes connecting the second city C_2 with the remaining cities is 19; but one of them is the route connecting C_1 with C_2. So the number of routes that connects two cities in which C_1 or C_2 is one of the cities is 19 + 18. Similarly, number of routes that connects two cities in which C_1, C_2 or C_3 is one of the cities is 19 + 18 + 17. Thus proceeding the answer to our question is $19 + 18 + 17 + 16 + \ldots + 3 + 2 + 1 = 190$.

4. Ram checks his purse and finds that he can buy an apple and three oranges or two apples for the money he has. I buy, from the same shop, two apples and two oranges for Rs.16. How much my friend should pay when he buys three apples and two oranges from the same shop?

Solution: Price of 1 apple and 3 oranges = price of 2 apples. Therefore, price of 3 oranges = price of 1 apple.
Hence, the price of 2 apples and 2 oranges = price of 6 oranges + price of 2 oranges = price of 8 oranges = Rs.16 (given).
So, price of an orange = Rs.2 and price of one apple = Rs.6.
The price for three apples and two oranges
$= 3\times$ Rs.6 $+ 2 \times$ Rs.2 =Rs.22.
Therefore, the amount the friend should pay = Rs.22.

5. Let $d(n)$ denote the number of divisors of a positive integer n. For example, $d(6) = 4, d(7) = 2, d(12) = 6$ as 1,2,3,6 are the divisors of 6; 1,7 are the divisors of 7; and 1,2,3,4,6,12 are the divisors of 12. We note that $\mathrm{d}(n) = 2$ if and only if n is a prime integer. Prepare a table which gives the values of $d(n)$ for $n = 1, 2, 3, ..., 20$

   (a) Find $d(4), d(49), d(121)$ and $d(37 \times 37)$.

   (b) Find n such that $1 \leq n \leq 100$ for which $d(n) = 3$.

   (c) Use the table you have prepared above to find n between 2000 and 2099 such that $d(n)$ is an odd number.

   (d) If $d(n)$ is a very big integer, then n is clearly a bigger integer. Looking in the opposite direction, if n is a very big integer can we say that $d(n)$ is at least half as big?

   (e) Can you find a big integer K such that for any integer n bigger than K we have $d(n) \geq 3$?

Solution:

Table for n and $d(n)$.

n	Div isors of n	$d(n)$
1	1	1
2	1,2	2
3	1,3	2
4	1,2,4	3
5	1,5	2
6	1,2,3,6	4
7	1,7	2
8	1,2,4,8	4
9	1,3,9	3
10	1,2,5,10	4
11	1,11	2
12	1,2,3,4,6,12	6
13	1,13	2
14	1,2,7,14	4
15	1,3,5,15	4
16	1,2,4,8,16	5
17	1,17	2
18	1,2,3,6,9,18	6
19	1,19	2
20	1,2,4,5,10,20	6

(a) From the table it is clear that for any square of a prime p the divisors are $1, p, p^2$ and hence $d(p^2) = 3$ for any prime p. This means that $d(4) = d(49) = d(121) = d(37 \times 37) = 3$.

(b) We see from the table that $d(n) = 3$ if and only if $n = p^2$ for some prime p. So we have to pick squares of prime numbers between 1 and 100. They are 4, 9, 25, 49.

(c) The table also shows that $d(n)$ is an odd number only when n is a perfect square (number like $1, 4, 9, 16, 25, 36, 49, \ldots$). So we have to pick perfect squares between 2000 and 2099. We note that $44 \times 44 = 1976, 45 \times 45 = 2025$ and $46 \times 46 = 2116$. So the only square number between 2000 and 2099 is 2025.

(d) **No.** By definition $d(n) = 2$ whenever n is a prime number and we have arbitrarily large prime numbers. In other words, as there is no biggest prime number, we have very large prime numbers n for which we always have $d(n) = 2$.

(e) **No, we cannot find.** As seen in the previous subdivision of the problem, we have bigger and bigger prime numbers n for which $d(n) = 2$.

6. Some prime numbers are generated as follows. Start with a prime number. For example 3. Then consider $2 \times 3 + 1$. It is 7.It is a prime number. Again multiply by 2 and add 1 to get $2 \times 7 + 1$ to get 15. Now 15 is not a prime. So find the least prime dividing 15; which is the number 3. The sequence generated so far is 3,7,3. If we continue this process we will get the sequence $3, 7, 3, 7, 3, 7, 3, 7, \ldots$. The process is given by

   (a) Start with a prime number p_1

   (b) Multiply p_1 by 2 and add 1 to get $2p_1 + 1$

   (c) If $2p_1 + 1$ is a prime write $2p_1 + 1 = p_2$

   (d) If $2p_1 + 1$ is not a prime call the smallest prime factor of $2p_1 + 1$ as p_2

   (e) Multiply p_2 by 2 and add 1 to get $2p_2 + 1$

   (f) If $2p_2 + 1$ is a prime write $2p_2 + 1 = p_3$

   (g) If $2p_2 + 1$ is not a prime call the smallest prime factor of $2p_2 + 1$ as p_3

(h) Continue this process to get a sequence of prime numbers $p_1, p_2, p_3, \cdots$

Starting with the prime number 11 and following the above procedure get the first 100 terms of the sequence; also find their sum.

Solution: If we start with 11 and proceed we get 11, 23, 47, 5, 11, 23, 47, 5, ... This sequence is repeating. It is called - (period 4) periodic. The terms 11,23,47,5 come in cycles. Therefore the 100th term in the sequence will be the fourth term of the 25th cycle; and it is 5. So the 100th term is 5. The first 100 terms are

$$11, 23, 47, 5, 11, 23, 47, 5, 11, 23, 47, 5, ..., 11, 23, 47, 5$$

The sum of the first 100 terms is

$$25 \times (11 + 23 + 47 + 5) = 25 \times 86 = 2150.$$

7. Consider the first five natural numbers 1,2,3,4,5. This set of five numbers is divided into two sets A and B where A contains two numbers and B contains the other three numbers. One example is $A = \{2, 4\}$ and $B = \{1, 3, 5\}$. How many such pairs A, B of sets are there?

   Solution: The different possible choices for A are

   $$\{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \{4,5\}.$$

   This is got by actual enumeration. We find that there are 10 different sets for A. Hence the number of pairs A, B of the required type is 10.

8. Draw a 4×4 square as shown. Fill the 16 squares with the letters a, b, c, d so that each letter appears exactly once in each

row and also exactly once in each column. Give at least two different solutions.

Solution: Write in any order a, b, c, d in a row(or column). Then cyclically change the letters in the other rows(or columns)

a	b	c	d
b	c	d	a
c	d	a	b
d	a	b	c

b	c	d	a
d	a	b	c
c	d	a	b
a	b	c	d

9. One can easily see that if a perfect square n^2 is divisible by a prime p then it is also divisible by p^2. For example any square integer that is divisible by 7 is also divisible by 49. Can a number written with 200 zeroes, 200 ones and 200 twos be a perfect square?

   Solution: Consider a number N written with 200 zeroes, 200 ones and 200 twos. The sum of the digits of this number (written in decimal scale) is $200 + 2(200) = 600$. So N is divisible by the prime number 3. But it is not divisible by 9. Hence no such N is a perfect square. The answer is NO.

10. The positive integers a and b satisfy $23a = 32b$. Can $a + b$ be a prime number? Justify your answer.

    Solution: $23a = 32b$. 32 does not divide 23. b should divide 23. In the other direction, 23 does does not divide 32, and so 23 should divide b. Thus b should divide 23 and 23 should divide b. $\therefore$ b should be 23. i.e., $23a = 32 \times 23$ making $a = 32$.

$\therefore a + b = 23 + 32 = 55 = 11 \times 5$

$\therefore a + b$ is not a prime number.

11. Each square in a 2 × 2 table is coloured either black or white. How many different colourings of the table are there?

Solution: 2 × 2 square is drawn on a fixed table and coloured either black or white.

4	3
1	2

Number the four square cells 1,2,3 and 4 in the anti clockwise direction. Taking it that the square cannot be rotated and can be turned upside down and so on and you are standing in front of the table. Then the number of ways these square cells can be coloured in 16 different ways.

However if the square cells can move (rotated and turned on sides) the answer is different. Colouring all the four square cells black or all the four cells white we get 2 different colouring

(1)

W	W
W	W

(2)

B	B
B	B

and these two different colouring do not change by rotating or turning.

(3) 3 white and 1 black – rotating about the centre thro 90°, 180°, 270°

(i)

B	B
W	B

(ii)

B	B
B	W

90°

(iii)

B	W
B	B

180°

(vi)

B	W
W	W

270°

Thus these 4 different colourings can be obtained from one way of colouring and hence taken as just one more way of colouring.

(4) Again colouring one cell white and three cells black gives one more way of colouring. Draw the corresponding figure.

(5) 2 White and 2 Black will look like the following.

W	W
B	B

— (a)

Again rotating, we get

W	B
W	B

⟶

B	B
W	W

⟶

B	W
B	W

So there are four ways of colouring is obtained from figure (a). So can be taken just as one way.

Final 2 Whites and 2 Black can be coloured diagonally.

(6)

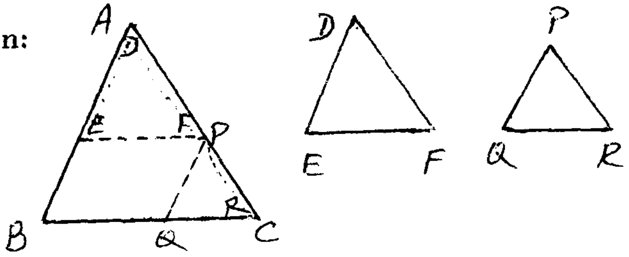
B	W
W	B

draw the diagrams obtained by rotating through $90^\circ, 180^\circ$ and 270°. How many distinct figures do you get?

Thus there are 6 different colouring if rotations (or turning or flipping are allowed) after colouring once.

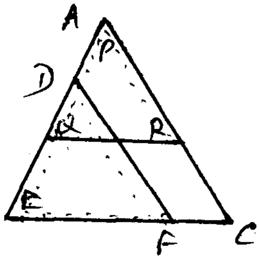
12. Explain why an equilateral triangle (a triangle with equal sides) cannot be covered by two smaller equilateral triangles.

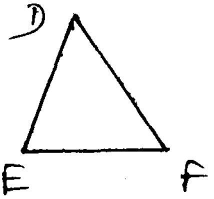
Solution:

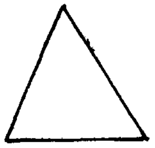


FIG

Let ABC be the given equilateral $\triangle$ and $\triangle PQR$ and $\triangle DEF$ are two smaller equilateral $\triangle$. Since the sides of $\triangle PQR$ and $\triangle DEF$ are less than the sides of $\triangle ABC$, after keeping $\triangle PQR$ and $\triangle DEF$ in two of the corners as shown in the figure, the quadrilateral region (in fact $BQPE$ is a $||^{gm}$) is not filled up. Whatever 2 vertices are chosen, a quadrilateral region can not be filled. and hence bigger equilateral $\triangle$ can not be completely covered by 2 small equilateral $\triangle^s$.





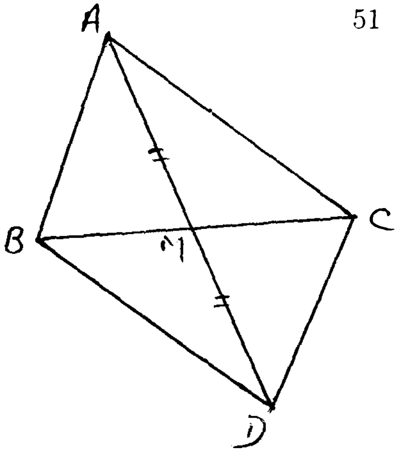


Some times, those smaller equilateral $\triangle$ kept inside the bigger equilateral $\triangle$ may overlap also leaving a quadrilateral region uncovered.

13. The side AC of a triangle is of length 2.7 cms., and the side AB has length 0.7 cms. If the length of the side BC is an integer, what is the length of BC?

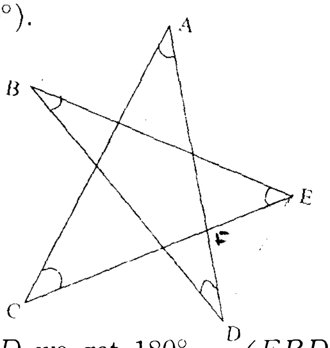
    Solution: The length of BC must be less than $AB + AC = 0.7 + 2.7 = 3.4$ cms. Also it must be greater than $AC - AB = 2.7 - 0.7 = 2$ cms. Therefore the only integral value possible for BC is 3cms. Answer: 3 cms.

14. It is well known that the diagonals of a parallelogram bisect each other. In any triangle the line segment joining a vertex with the midpoint of the opposite side is called a median. If ABC is any triangle prove that the sum of the lengths of the three medians is not greater than the triangle's perimeter.



Solution: Let M be the midpoint of BC. Produce AM to D so that $AM = MD$. We see that $ABCD$ is a parallelogram. From the figure it is clear that twice the median through $A = 2AM = AD \leq AB + BD = AB + AC$. Similarly twice the medians through B and C are in turn less than or equal to $BA + BC$ and $CA + CB$ respectively. Adding we see that the sum of the lengths of the three median is not greater than $\frac{1}{2}(AB + AC + BC + BA + CA + CB) = AB + BC + CA =$ perimeter of the triangle ABC. Hence the result.

15. In the figure we have a star with five vertices A,B,C,D.E. Find the sum of the angles at the vertices A,B,C,D,E of the five pointed star.(You may use the fact that in any triangle the sum of the angles is 180°).



Solution: From ΔBED we get $180^\circ = \angle EBD + \angle BDE + \angle BED = \angle B + \angle D + \angle E + \angle CED + \angle EDA = \angle B + \angle D + \angle E + 180^\circ - \angle CFA = \angle B + \angle D + \angle E + \angle A + \angle C$. Thus

$$\angle A + \angle B + \angle C + \angle D + \angle E = 180^\circ$$

where F is the point of intersection of AD and CE.

16. Observe the sequence 9, 91, 19, 911, 191, 119, 9111, 1911, 1191, 1119 $\cdots$. What is the 45th term of the sequence?

Solution: In the given sequence we notice that there are r members with r digits, $r = 1, 2, \cdots$. Since $1 + 2 + \cdots + 9 = \frac{9 \times 10}{2} = 45$, the 45th member of the sequence is the ninth number of nine digited numbers. In the nine digited numbers the number 9 shifts from the left most digit to the right most digit successively with the remaining digits all equal to 1. Hence the required 45th member of the sequence is 111111119. (Eight 1's followed by 9)

17. How many three digit numbers are there with no *'zero'* present in it? (Numbers like $203, 460$ and $600 \cdots$ contain *'zero'* in them and should not be counted.)

Solution: Three digit numbers are $100, 101, \cdots, 999$, totally 900. For every non-zero digit in the hundred's place there can be nine possible non-zero entry in the ten's place with unit's filled by 0.

Thus there are $9 \times 9 = 81$ numbers with zero only in the units place. Similarly there are $9 \times 9 = 81$ numbers with zero only in the ten's place. there are just 9 numbers $(100, 200, \cdots, 900)$ with 0's in the ten's and unit's place. Thus the number of numbers with 0's among the three digited numbers is $81 + 81 + 9 = 171$. Hence the number of numbers without zeros is $900 - 171 = 729$.

Aliter:

If zero is not to be found in a three digited number we can fill hundred's place in 9 ways and for every such way in 9 ways the ten's place and again for every possibility of filling the hundred's and ten's place the units place can be filled in 9 ways. Thus the total number of ways in which a three digited number can be formed without the digit 0 is $9 \times 9 \times 9 = 729$.

18. If we multiply 2 by itself repeatedly four times we get $2 \times 2 \times 2 \times 2 = 16$ and its unit digit is 6. Suppose we multiply 2 with itself repeatedly 2005 times we get a big number B. What is the unit digit of B?

Solution: Note that $2^4 = 16$. Also we know that any two numbers with 6 in the unit place when multiplied give another number with 6 in the unit's place.

Now $2004 = 4 \times 501$. Hence $2^{2004} = (2^4)^{501}$ is a number ending with 6. Multiplying this number by 2 we get 2^{2005} and this has 2 in the unit's place.

Aliter:

$2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32, 2^6 = 64, \cdots$ form a sequence of numbers with the digits in the unit place forming the sequence.

$$2, 4, 8, 6, 2, 4, 8, 6, 2, 4, \cdots .$$

Thus $1st, 5th, 9th, \cdots, (4n+1)th, \cdots$ members of this sequence are all 2, for $n = 0, 1, 2, \cdots 2005th$ member is $(4 \times 501 + 1)$th member and corresponds to $n = 501$. Hence the number in the unit's place of 2^{2005} is 2.

19. $\boxed{3|6}$ $\boxed{3}$ $\boxed{1}$ and $\boxed{6|1}$ are four blocks.

Using these four blocks how many six digit numbers can be formed?

Solution: Denote the blocks, $\boxed{3|6}$ $\boxed{3}$ $\boxed{1}$ and $\boxed{6|1}$ by A, B, C and D respectively. The required numbers are of the form $ABCD, ADCB, DABC, \cdots$. The left most block can be

(For example 26 has four divisors - 1,2,13 and 26). Hence the answer is all square numbers 1 to 100: −4,9,16,25,36,49,64,81.

23. If x is a number between 0 and 10 (for example x can be 1.01 or 0.75 or 2.08) and y is the number $x + \frac{1}{x}$ then can y be bigger than 2005? Explain your answer.

    Solution: If $1 \leq x \leq 10, x + \frac{1}{x}$ lies between 2 and 10.1.

    Hence y cannot be greater than 2005 for $1 \leq x \leq 10$.

    When $x \geq \frac{1}{2004}$, $y = x + \frac{1}{x} < 2005$.

    However, if $x \leq \frac{1}{2005}, y = x + \frac{1}{x} > 2005$.

    Thus for all $0 < x \leq \frac{1}{2005}$, $y = x + \frac{1}{x} > 2005$.

24. The difference between two positive numbers is 16. If the smaller of these two numbers is $\frac{3}{5}$ of the larger, what is the value of the smaller number?

    Solution: The numbers can be taken as $3x$ and $5x$ (both positive, for some x). It is given that $5x - 3x = 2x = 16$. Hence $x = 8$. Thus the numbers are 24 and 40.

25. Two people were hired for a job for a total of Rs.45. If they completed the job with one person working for 1 hour and 20 minutes and the other working for 40 minutes and if they share the wages of Rs.45 in proportion to the amount of time each person worked, how much did the person who worked longer receive?

    Solution: The persons work for 80 and 40 minutes respectively. So they have to share the amount of Rs. 45 in the ratio 80 : 40 or 2 : 1. Hence the person working for a longer time will get Rs.30 and the other will get Rs. 15.

26. If a and b are positive integers each greater than 1, and if $11(a-1) = 13(b-1)$ what is the least possible value of $a+b$?

    Solution: As 11 and 13 are coprimes $11(a-1) = 13(b-1)$ implies that $(b-1)$ is a multiple of 11 and $(a-1)$ is a multiple of 13. As a, b are both greater than one the least possible values of a and b are respectively 14 and 12. Hence $a+b = 26$ is the required answer.

27. If the average of $16, 10$ and n is between 18 and 21 what is the greatest possible value of n?
    (By the average of a, b, c we mean the number $\frac{a+b+c}{3}$).

    Solution: Since the average is between 18 and 21, it should be 19 or 20.

    To get the greatest value of n, the average should the greatest possible value which is 20.

    That is average of 3 numbers is 20.
    The sum of these three numbers is $3 \times 20 = 60$.
    Sum of two of them is $16 + 10 = 26$.
    So the third number n is $60 - 26 = 34$.
    i.e., the greatest value of n is 34.

28. In the set of natural numbers $1, 2, 3, \cdots$ we define a new multiplication as follows: For positive integers m, n we find the usual product mn, divide mn by 7 and find the remainder k, and we define $m \star n = k$. For example $102 \star 8 = 4$ since $102 \times 8 = 816 = 7 \times 116 + 4$. Again $84 \star 5 = 0$ since 84×5 is a multiple of 7 and the product leaves 'zero' remainder when divided by 7.

    Now, find an integer k such that $2005 \star k = 1$ in our new multiplication '$\star$'.

Solution: $2005 \star k = 1$ implies that $2005k - 1$ is a multiple of 7. i.e., $(7 \times 286 + 3)k - 1$ is a multiple of 7 i.e., $3k - 1$ is a multiple of 7. The least integral value of k is 5. The other values of k are given by $7m + 5 (m \geq 1)$. Thus $2005 \star k = 1$ for $k = 5, 12, 19, 26, \cdots$ (an arithmetic progression with first member 5 and common difference 7).

29. Ram's television has channels 2 to 39. If he starts on channel 13 and surfs pushing the channel-up button 407 times, on what channel is the television when he stops?

Solution: As there are 38 channels in the television, starting from any channel by pushing up 38 times one comes back to that channel. Starting from channel 13 pushing up 407 more times is equivalent to pushing up 407 minus the greatest multiple of 38 less than 407 i.e. 27 more times. Thus one will reach channel 2 at the end of 407 pushing ups.

30. Find the number of times the hour hand and the minute hand of a clock form a right angle with each other between 06.00 a.m. and 12.00 noon on the same day.

Solution: During 6 a.m. to 7 a.m. the minute hand will cross the hour hand once and at some time before and at some time after this event the angle between them will be 90°.

A similar thing holds for every hour until 12 noon. However at 9 a.m. the angle between the two hands is 90° and this would have been counted once between 8.00 a.m. and 9.00 a.m. and again between 9.00 a.m. and 10 a.m. Taking account of this the number of times the angle between the hands is 90° is $2 \times 6 - 1 = 11$.

31. Krishna was given the task of removing the multiples $2, 3, 5, 7$ from the set of integers from 1 to 100. Find the number of numbers that remain.

    Solution: If $2, 3, 5, 7$ appear in the prime number representations of integers between 1 and 100, they get automatically removed. Only the first power of the other primes ≤ 100 will be found in the prime representation as $11^2, 13^2, \cdots$ are all greater than 100 and so are $11 \times 13, 13 \times 17, \cdots$ greater than 100.

    Thus the numbers that remain are 1 and all 2 digit primes. These are the twenty two numbers 1,11,13,17,19,23,29,31,37, 41,43,47,53,59,61,67,71,73,79,83,89 and 97.

32. Shiv was asked to remove numbers, which have an even number of divisors from the set of integers from 1 to 150. If Shiv performs his task correctly what are the numbers that remain after Shiv completes his job? (For example, 10 has 4 divisors, namely $1, 2, 5, 10$).

    Solution: Every perfect square from 1 to 150 has an odd number of divisors. All other numbers will have an even number of divisors. For example, 64 has 1,2,4,8,16,32 and 64 (seven) as divisors and 102 has $1, 2, 3, 6, 17, 34, 51$ and 102 as divisors (eight in number). Thus there will remain 12 numbers viz $1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121$ and 144 after Shiv completes the job.

33. If we are given 4 match sticks having lengths 1cm, 2cm, 3cm, 4cm, how many different triangles can be made choosing 3 sticks at a time.

    Solution: If 1 cm long match sticks were taken to form a side with any two of the remaining sticks no triangle can be formed.

This is because for any such choice we have at least one side greater than the sum of the other two. Hence we are left with only 3 match sticks of lengths 2cm, 3cm, and 4cm. which will form just one triangle.

34. The sum of the first n odd numbers is 2304. Find n.

Solution: We know that

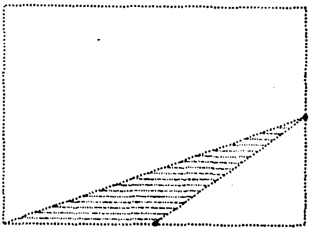
$1 + 2 + 3 + \cdots + 2n = \frac{2n(2n+1)}{2} = n(2n+1)$ and

$2 + 4 + \cdots + 2n = 2(1 + 2 + \cdots + n) = n(n+1)$.

$$\begin{aligned} \therefore\ 1 + 3 + \cdots + (2n-1) &= \text{sum of first } n \text{ odd numbers} \\ &= n(2n+1) - n(n+1) = n^2 \end{aligned}$$

As $2304 = 48 \times 48$, we see that the sum of first 48 odd numbers is 2304. i.e., $1 + 3 + \cdots + 95 = 2304$.

35. In the rectangle given below, the indicated triangle has area 7 cm^2. Find the area of the rectangle. The thick dots in the diagram are midpoints of the corresponding sides.



Solution: Let the length of the rectangle be $2a$ cm and the breadth be $2b$ cm. Then area of the rectangle $= 4ab$ cm^2.

The shaded area $= \frac{1}{2} \times 2a \times b - \frac{1}{2}ab = \frac{ab}{2}$.

This is given to be 7 cm^2. $\therefore ab = 7 \times 2 = 14$.

Hence area of the rectangle $= 4 \times 14 = 56$ cm^2.

36. In a pair of dice one die has 3 ones and 3 fours on its faces. What should be the numbers on the faces of the other die so that the possible sums of the numbers shown on the dice are from 1 to 12 when both are rolled?

[For example if one die has 0,0,0,1,1,1 on its faces and the other die has 1,3,5,7,9,11 then some of the possible sums are 0+1=1, 1+7=8, 1+11=12 etc.]

Solution: Given one die has faces $1, 1, 1, 4, 4, 4$. We require faces on second die to get sums from 1 to 12.

Having '0' in the 2^{nd} die, we get sums $1 + \underline{0} = 1$ and $4 + \underline{0} = 4$. Likewise, we can construct a table as follows:

First die has	If 2^{nd} die has	sums obtained
1,4	0	1,4
1,4	1	2,5
1,4	2	3,6

First half is obtained

First die has	If 2^{nd} die has	sums obtained
1,4	6	7,10
1,4	7	8,11
1,4	8	9,12

Second half is obtained.

$\therefore$ The second die should have faces $0, 1, 2, 6, 7, 8$.

37. Build a sequence starting with the first term $t_1 = 1$ the second term $t_2 = 2$ and the nth term $t_n = \frac{t_{n-1}+1}{t_{n-2}}$ $n \geq 3$. [Example $n = 3$ gives t_3 the third term as $t_{n-1} = t_{3-1} = t_2$ $t_{n-2} = t_{3-2} = t_1$ $t_3 = \frac{t_2+1}{t_1} = \frac{2+1}{1} = 3$] Find the first seven terms and hence find the sum of the sequence to 100 terms.

Solution:

$\boxed{t_1 = 1}$ (given)

$\boxed{t_2 = 2}$ (given)

$$t_3 = \frac{t_2+1}{t_1} = \frac{2+1}{1}. = 3 \text{ (or) } \boxed{t_3 = 3}$$

$$t_4 = \frac{t_3+1}{t_2} = \frac{3+1}{2} = 2 \text{ (or) } \boxed{t_4 = 2}$$

$$t_5 = \frac{t_4+1}{t_3} = \frac{2+1}{3} = 1 \text{ (or) } \boxed{t_5 = 1}$$

$$t_6 = \frac{t_5+1}{t_3} = \frac{1+1}{2} = 1 \text{ (or) } \boxed{t_6 = 1}$$

$$t_7 = \frac{t_6+1}{t_5} = \frac{1+1}{1} = 2 \text{ (or) } \boxed{t_7 = 2}$$

Note that t_6 and t_7 are same as t_1 and t_2 respectively. Thus the sequence has groups of repeated terms.

$\therefore$ the sequence is

$$\underbrace{1,2,3,2,1,}\ \underbrace{1,2,3,2,1,}\ \underbrace{1,2,3,2,1,}\cdots.$$

$$\begin{aligned}\therefore \text{ Sum of first 100 terms } &= \frac{100}{5} \times (1+2+3+2+1) \text{ i.e.,}\\ &\quad 20 \text{ groups of } (1+2+3+2+1)\\ &= 20 \times 9\\ &= 180.\end{aligned}$$

38. Find the number of days in a year in each month which are the multiples of their month number, specifying such dates in each month.

[Example 20^{th} of October is divisible by the month number of October which is 10.]

Solution: Construct a table as follows:

Month	Month number	No. of days in the month	Dates which are multiples of month number	Number of counts
Jan	1	31	1,2,3,· · ·, 31	31
Feb	2	28/29	2,4,6,· · ·, 28	14
Mar	3	31	3,6,9,· · ·,30	10
Apr	4	30	4,8,12,· · ·, 28	7
May	5	31	5,10,15,· · ·, 30	6
June	6	30	6,12,18,24,20	5
July	7	31	7,14,21,28	4
Aug	8	31	8,16,24	3
Sept	9	30	9,18,27	3
Oct	10	31	10,20,30	3
Nov	11	30	11,22	2
Dec	12	31	12,24	2
			Total	**90**

$\therefore$ The number of days in a year such that the dates which are the multiples of their month number is 90.

39. Find all positive even integers less than 50, which can be written as a sum of three consecutive integers and find the sum of all these integers.

Solution: The sum of even, odd, even is odd and sum of odd, even, odd is even.

To get an even sum, the three consecutive integers are odd. even, odd (or) should start with odd.

$$\begin{array}{lll} 1+2+3 = 6 = 1\times 6. & 9+10+11 = 30 = 5\times 6 \\ 3+4+5 = 12 = 2\times 6. & 11+12+13 = 36 = 6\times 6 \\ 5+6+7 = 18 = 3\times 6. & 13+14+15 = 42 = 7\times 6 \\ 7+8+9 = 24 = 4\times 6. & 15+16+17 = 48 = 8\times 6 \end{array}$$

Thus the positive integers satisfying the conditions are of the form $6\times n$. where n is the rank of the starting odd number.

Thus 1 is the first odd number $1+2+3 = 1\times 6$. 3 is the second odd number (i.e., $2\times 2-1$). The sum is $3+4+5 = 12 = 2\times 6$ and again $15 = (2\times 8-1)$ is the eighth odd number and $15+16+17$ is $8\times 6 = 48$.

In general, if the three consecutive numbers are $2n-1, 2n, 2n+1$, their sum is clearly $6n$.

The sum all these numbers is

$(3+4+5)+(5+6+7)+\cdots+(15+16+17) = 6+6\times 2+6\times 3+\cdots+6\times 8 = 6[1+2+3+\cdots+8] = \frac{6\times 8\times 9}{2} = 216$.

40. Find all four digit numbers, where middle two digits are 1,3 in that order such that the number is exactly divisible by 24 [i.e is an exact multiple of 24]

Solution: Let the four-digit number be A 1 3 B.

The number is divisible by 24

⇒ The number is divisible by **3** and **8**.

Test of divisibility for 8 is 'Last – 3 digit number should be divisible by 8'.

Only 136 is divisible by 8 in one hundred and thirties.

$\therefore \boxed{B = 6}$ and Number is A 1 3 **6**.

Test of divisibility for 3 is 'sum of the digits is divisible by 3'.

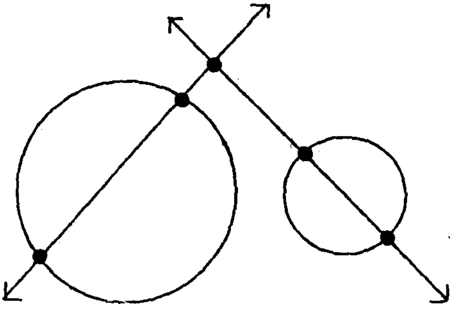
Hence $\boxed{A = 2 \text{ or } 5 \text{ or } 8.}$

$\therefore$ 4-digit numbers are 2136, 5136, 8136.

41. Draw atleast 5 possible diagrams with two straight lines and two circles to get 5 points of intersections; the points of intersections may be, line and line, circle and circle or line and circle.

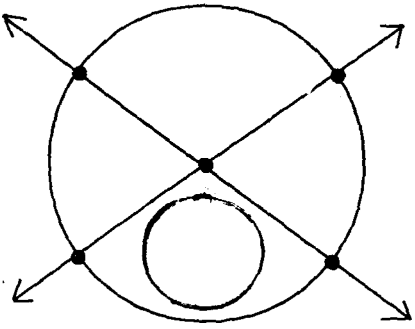
Solution: Some possible diagrams are:

A.



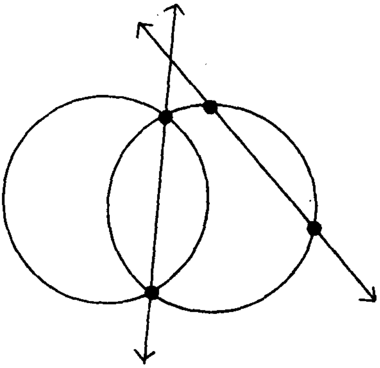
Non-intersecting unequal circles, non-parallel lines.

B.



Non-parallel lines, one circle inside other & do not intersect the lines.

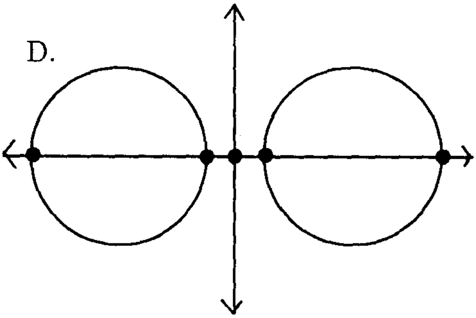
C.



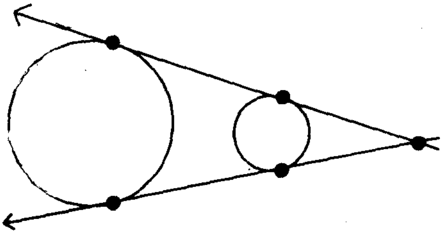
Intersecting circles, line intersecting lines and circle but not other circle change.

Two lines are intersecting one line is drawn through the points of intersection of the 2 circles. Second line is drawn to intersect one of the two circles and intersected the first line. Two circles having no common point one line cuts both the circles, second cut only the first line.

D.



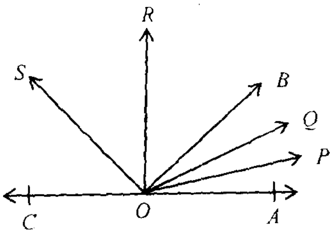
E.

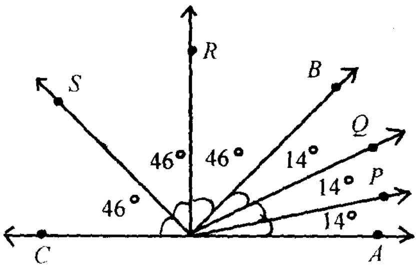


Lines touching the circles and cutting each other.

There are some more possibilities.

42. In the adjoining figure, AOC is a straight line, and $\angle AOB = 42^0$. OP and OQ trisect angle AOB [which means that they divide the angle into three equal parts.] OR and OS trisect angle $\angle BOC$. Showing all working calculate the angle $\angle QOR$ and $\angle POS$.





Solution:

$\angle AOB = 42°$; $\angle AOP = \angle POQ = \angle QOB = \frac{42}{3} = 14°$

$\angle BOC = 180° - \angle AOB = 180° - 42° = 138° =$

$\angle ROB = \angle ROS = \angle SOC = \frac{138}{3} = 46°$

$m\angle QOR = m\angle QOB + m\angle BOR = 46° + 14°$

i.e., $m\angle QOR = 60°$.

$m\angle POS = 14° + 14° + 46° + 46° = 120°$

Note: $\overrightarrow{OP}, \overrightarrow{OQ}$ trisects $m\angle BOA = 42°$.

$\therefore m\angle SOC = m\angle ROS = m\angle BOR = \frac{138°}{3} = 46°$.

$\boxed{\therefore m\angle QOR = 60°}$ $\boxed{m\angle POS = 120°}$

43. $A_1 . A_2, A_3, A_4, A_5, A_6, A_7$ are 7 collections of natural numbers such that

(a) Average of A_1 = Average of $A_2 = \cdots =$ Average of A_7

(b) A_1 contain 2 numbers, A_2 contain 3 numbers, $\cdots$ and A_7 contains 8 numbers

(c) All the collection $A_1, A_2, \cdots A_7$ contain the two numbers 4 and 6.

(d) Each collection contains distinct numbers. Find all the collections.

Solution: Given:

Group	Number of elements	Contains
A_1	2	$\{4,6\}$
A_2	3	$\{4,6,a\}$
A_3	4	$\{4,6,b,c\}$
A_4	5	$\{4,6,d,e,f\}$
A_5	6	$\{4,6,k,l,m,n\}$
A_6	7	$\{4,6,p,q,r,s,t\}$
A_7	8	$\{4,6,u,v,w,x,y,z\}$

No group contains same number twice.

Group $\quad A_1 = \{4,6\}. \qquad (1)$

Average of $A_1 = 5$.

$A_1 = (4,6)$. Average of $A_1 = 5$. Sum of the numbers in $A_1 = 10$ since $A_2, A_3, A_4 \cdots A_7$ have the same average, sum of the numbers in these groups are $3 \times 5 = 15$, $4 \times 5 = 20, \cdots 8 \times 5 = 40$ and hence

$A_2 = \{4, 6, 5\}$ and

$A_3 = \{4, 6, b, c\}$ and $b + c = 10$ (why?). One such collection is $\{4, 6, 1, 9\}$ [Find other possible collections].

$A_4 = \{4, 6, d, e, f\} \Rightarrow d + e + f = 15$, one such collection is $\{4, 6, 1, 2, 8\}$.

$A_5 = \{4, 6, k, l, m, n\}$ and $k + l + m + n = 20$. The answer for this is $\{4, 6, 3, 7, 1, 9\}$. Similarly,

$A_6 = \{4, 6, p, q, r, s, t\}$ and $p + q + r + s + t = 25$ gives A_6 as $\{4, 6, 2, 5, 6, 9, 3\}$.

$A_7 = \{4, 6, u, v, w, x, y, z\}$ and $u + v + w + x + y + z = 30$ gives A_7 as $\{4, 6, 1, 2, 3, 5, 8, 11\}$.

Note that every collection has 4, 6 as a common pair of numbers and no other number is common number for **all the 7 groups**. Within the groups, all numbers are distinct.

44. Two four digit numbers *abcd* and *efgh* are constructed using all the digits from 1 to 8. [only a, b, c, d, e, f, g and h are the digits of the two four digit numbers]. It is found that $abcd \times 2 = efgh$. Find the numbers.

Solution: Given $abcd \times 2 = efgh$. And $abcd, efgh$ are 4 digit numbers constructed from digits 1 to 8.

None of the values of a, b, c, d, e, f, g, h can be 0 or 9.

As it is not mentioned that the digits are different (or) all the digits are used, it should be noted that digits may repeat.

In such case, there will be plenty of solutions. The smallest such *abcd* is 1111. Its corresponding *efgh* is 2222. The largest *abcd* will be 4444. Its corresponding *efgh* is 8888.

Only certain 4-digit numbers from 1111 to 4444 satisfy $abcd$ and we have the respective $efgh$'s.

The conditions are:

(i) $d \neq 5$. Otherwise $h = 0$.

(ii) $cd \neq 46, 47, 48$. Otherwise $g = 9$.

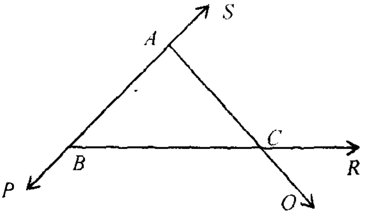
(iii) $cd \neq 51, 52, 53, 54$. Otherwise $g = 0$.

(iv) $bcd \neq 456$ to 554 otherwise $f = 9$ or f (or) $g = 0$.

(v) $a \neq 5, 6, 7, 8$. Otherwise $efgh$ will not be 4-digit number.

One solution where all a, b, c, d, e, f, g, h are different is $(1764) \times 2 = (3528)$.

45. In the adjoining figure $\angle CBP + \angle BCQ = 250^0$, $\angle ACR + \angle CAS = 230^0$ Determine the angles of the triangle ABC.



Solution: Given

$$\angle CBP + \angle BCQ = 250^\circ \tag{1}$$

$$\angle ACR + \angle CAS = 230^\circ \tag{2}$$

We have $\angle CBP + \angle ABC = 180^\circ$ (Linear pair angles are supplementary).

Similarly, $\angle BCQ + \angle ACB = 180^\circ$.

Adding the above two equations, we get

$$(\angle CBP + \angle BCQ) + \angle ABC + \angle ACB = 360^\circ$$

or $250^\circ + \{\angle B + \angle C(\text{ of } \triangle ABC)\} = 360^\circ$ from (1).

$\therefore \angle B + \angle C = 360^\circ - 250^\circ = 110^\circ$.

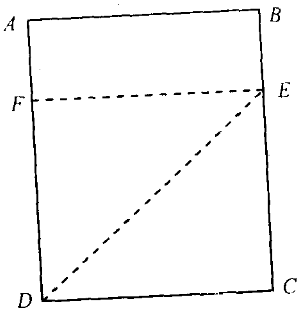
But $\angle A + \angle B + \angle C = 180^\circ \Rightarrow \angle A = 70^\circ$.

$\angle A = 70^\circ \Rightarrow \angle CAS = 110^\circ \Rightarrow \angle ACR = 120^\circ$ from (2).

$$\angle ACR = 120^\circ \Rightarrow \angle C = 60^\circ.$$

$\therefore \angle B = 50^\circ$. [180 − (70 + 60)]

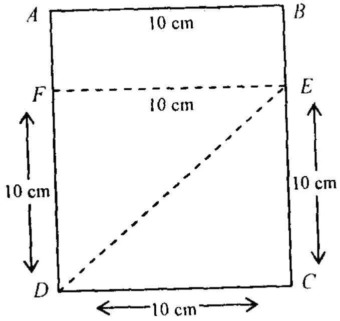
46. $ABCD$ is a rectangular sheet of paper. The corner C is made to fall on F along AD by folding the paper through the corner, diagonally so that a figure $ABCDEFA$ is got, i.e. a rectangle $ABEF$ together with the triangle FED is got. AB = 10cm. If the area of the rectangle $ABEF$ is equal to area of the triangle FED, find the length AD of the rectangle and find the perimeter and area of the rectangle $ABCD$. [You may know how to make a paper boat from a rectangular sheet of paper]



Solution: $EF = EC = 10$cm (due to folding). Now area of $\triangle EFD = \frac{1}{2}\times$ Area of square $CDFE$

$$= \frac{1}{2} \times 10 \times 10\text{cm}^2 = 50\text{cm}^2$$

Given Area of rectangle $ABEF$ =Area of $\triangle EFD$.



$\therefore$ Area of rectangle $ABEF$

$$= 10 \times AF = 50\text{cm}$$

$\Rightarrow$ $AF = 5\text{cm}$ or $BE = 5\text{cm}$ (opposite sides of rectangle are equal)

Area of rectangle $ABCD = 10\text{cm} \times 15\text{cm} = 150\text{cm}^2$

Perimeter of rectangle
$ABCD = 15\text{cm} + 10\text{cm} + 15\text{cm} + 10\text{cm} = 50\text{cm}$.

47. Find all values of natural number n, for which $\frac{630}{2n-1}$ is also a natural number

Solution: Given $\frac{630}{2n-1}$ should be a natural number.

For 'n' being natural.

$(2n - 1)$ should be an odd factor of 630.

As $630 = 2 \times 3^2 \times 5 \times 7$, the odd factors of 630 are $1, 3, 5, 7, 9, 15, 21, 35, 45, 63, 105, 315$.

2n-1	1	3	5	7	9	15	21	35
n	$\frac{1+1}{2} = 1$	$\frac{3+1}{2} = 2$	$\frac{5+1}{2} = 3$	4	5	8	11	18

2n-1	45	63	105	315
n	45+1/2=23	32	53	158

$\therefore$ Values of 'n' are $1, 2, 3, 4, 5, 8, 11, 18, 23, 32, 53$ and 158.

48. Given five points A, B, C, D and E as shown in the figure, points A, B, C lying on the same line. find the number of triangles whose vertices are from A, B, C, D and E.

A • B • C •

D • E •

Solution: Points A, B, C are in the same line

A triangle can be formed using the given points in 2 ways.

A • B • C • ← 1^{st} row

D • E • ← 2^{nd} row

2 points from 1st row	1 point from 2nd row	&	2 points from 2nd row	1 point from 1st row
$\triangle ABD, \triangle ABE$				
$\triangle ACD, \triangle ACE$			$\triangle DEA, \triangle DEB, \triangle DEC$	
$\triangle BCD, \triangle BCE$				

$\therefore$ 9 triangles are possible.

49. Using only the digits 3,6 and 9, find all four digit numbers using each of the digits atleast once, so that all the numbers thus got are divisible be 18.

Solution: Let the 4 digits be $3, 6, 9$ and x where x can be 3 or 6 or 9 only. Given that the 4 digit number is divisible by 18.

$\therefore$ The number is divisible by 9 and 2. This is possible only if the last digit is 6. Since 6 is the only even digit among $3, 6, 9$. Sum of digits should be divisible by 9.

i.e., $(3 + 9 + x + 6)$ is divisible by 9. This is possible only if $x = 9$. $\therefore$ The 4-digit numbers are 9936, 9396, 3996.

50. Amith, Bharath, Chithra and Deepa are friends and one of them committed a crime. They made the following statements. If only one of the statements is true, find who was guilty.

Amith: Bharat did it; *Bharath:* Deepa did it,
Chithra: I did not do it; *Deepa:* Bharat lied when he said I did it.

Solution: Given exactly one is criminal & one is true and rest are false.

Statement by	If True, then truth is	If False, then truth is
A	B is Criminal	B is not criminal
B	D is criminal	D is not criminal
C	C is not criminal	C is criminal
D	D is not criminal	D is criminal

where $A \rightarrow$ Amit, $B \rightarrow$ Bharath, $C \rightarrow$ Chitra $D \rightarrow$ Deepa.

From Table

(i) A cannot be true because if A is true, then B, C, D are false $\Rightarrow$ B and C are criminals since there cannot be more than 1 criminal. Also D cannot be criminal and not criminal at the same time.

(ii) B cannot be true otherwise there are 2 criminal in D, C which is not so.

(iii) C cannot be true otherwise D is criminal and not criminal at the same time, which is impossible.

(iv) D can be true implying B is not criminal,

(A-False)

D is not criminal, C is criminal and D is

(B-False) (C-False)

not criminal $\Rightarrow$ $\boxed{\text{Chitra is the criminal.}}$

(D-true)

51. Find all natural numbers which divide 2007 and leave a remainder 207.

Solution: The factors of $2007 - 207 = 1800$ which are > 207 are the required natural numbers.

They are $\frac{1800}{1}, \frac{1800}{2}, \frac{1800}{3}, \frac{1800}{4}, \frac{1800}{5}, \frac{1800}{6}, \frac{1800}{8}$.

$\therefore$ The required numbers are

$$1800, 900, 600, 450, 360, 300 \text{ and } 225.$$

52. The sum of 27 natural numbers is 30. Find the maximum product of these numbers. [Hint: There are more than one set of such numbers.]

Solution: In case, all the 27 numbers are natural numbers, the product is maximum, when the number of 1's in the 27 numbers is minimum. As $23(1) + 4(2) = 31 > 30$ [23 ones and 4 twos, the number of 1's cannot be < 24.

$\therefore$ **Minimum number of 1's is** 24.

$\therefore$ 24 ones and 3 twos gives 30 and the number of numbers is 27

The product will be $\underbrace{1 \times 1 \times 1 \times \cdots \times 1}_{24 \text{ times}} \times \underbrace{2 \times 2 \times 2}_{3 \text{ times}} = \boxed{8}$ whic

is maximum.

Alternate way is

number of 1’s	27	26	25	24
number of 2’s	–	1	2	3
product	1	2	4	8.

But there should be tabulation for number of 3’s number of 4’s along with number of 1’s and 2’s. e.g.

number of 1’s	no of 2’s	no of 3’s	no of 4’s	product
25	1	1		6
26	-	-	1	4

etc.,

Note: But out of the products 1,2,4,6 and 8, 8 is the maximum.

53. Find all four digit numbers formed by using the digits 2, 5 and 6, using each digit at least once and such that the four digit numbers thus formed are divisible by 3.

Solution: Given that the digits 2, 5, 6 should be used and all the digits should be used to form a 4-digit number. Further, the 4-digit number should be divisible by 3.

$\therefore$ The digits used to form 4-digit number are $\boxed{2,5,6,2}$

or $\boxed{2,5,6,5}$

$\therefore$ The 4-digit numbers are

2256	2526	2562	5226	5262	5622
2265	2625	2652	6225	6252	6522

and

5526	5256	5265	2556	2565	2655
5562	5652	5625	6552	6525	6255

There are 24 numbers satisfying the given conditions.

54. In the addition sum beside, different letters stand for different digits. Find what each letter stands for. Find at least two solutions

```
NET
TEN
---
END
---
```

Solution: $N + T < 10$. Otherwise, the result is a 4-digit number. $\therefore$ There's no carry over in the ten's place value. As E, D are different, there must be a carry over '1' in the hundred's place value. (The sum of single digits is maximum 18 and cannot be a carry over > 1).

```
 N    E     T         N E T
+T   +E    +N        +T E N
--   ----  --        ------
 E   10+N   D         E N D
--   ----  --        ------
```

$$N + T + 1 = E;\ E + E = 10 + N;\ T + N \doteq D$$

$$N + T + 1 + N + T + 1 = 10 + N \Rightarrow \boxed{N + 2T = 8}\ .$$

$\therefore$ N must be even.

Let $\boxed{N=2,\ T=3} \Rightarrow \boxed{D=5} \Rightarrow \boxed{E=6}$ i.e. (1)

$$\begin{array}{r} 2\ \ 6\ \ 3 \\ +3\ \ 6\ \ 2 \\ \hline 6\ \ 2\ \ 5 \\ \hline \end{array}$$

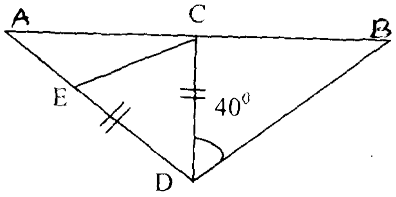
Let $\boxed{N=4,\ T=2} \Rightarrow \boxed{D=6,\ \ E=7}$ i.e. (1)

$$\begin{array}{r} 4\ \ 7\ \ 2 \\ +2\ \ 7\ \ 4 \\ \hline 7\ \ 4\ \ 6 \\ \hline \end{array}$$

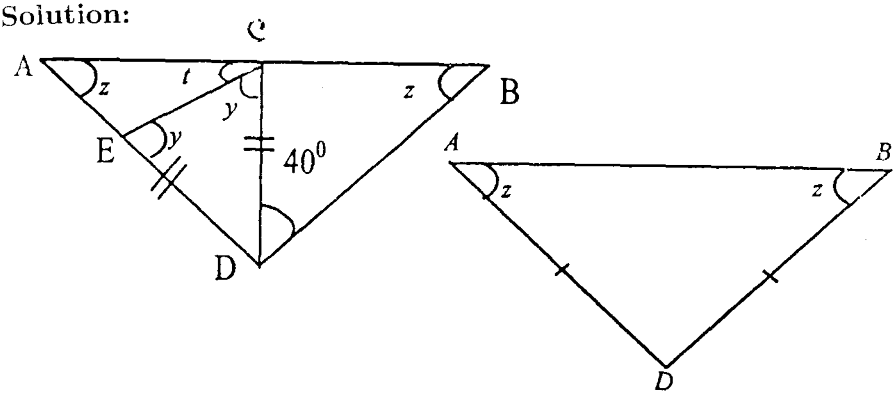
Also $\boxed{N=6,\ T=1} \Rightarrow \boxed{D=7,\ E=8} \Rightarrow$ (1)

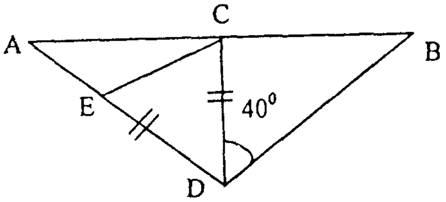
$$\begin{array}{r} 6\ \ 8\ \ 1 \\ 1\ \ 8\ \ 6 \\ \hline 8\ \ 6\ \ 7 \\ \hline \end{array}$$

55. In the adjoining figure find the size of $\angle ACE$, given $AD = DB$ and $DE = DC$.



Solution:





Let the angle measures be indicated as above.

By exterior angle property of $\angle C$ in $\triangle BDC$,

$$\boxed{t + y = 40^\circ + z} \qquad (1)$$

By exterior angle property angle measure

$$\angle CED = \angle ECA + \angle CAE$$

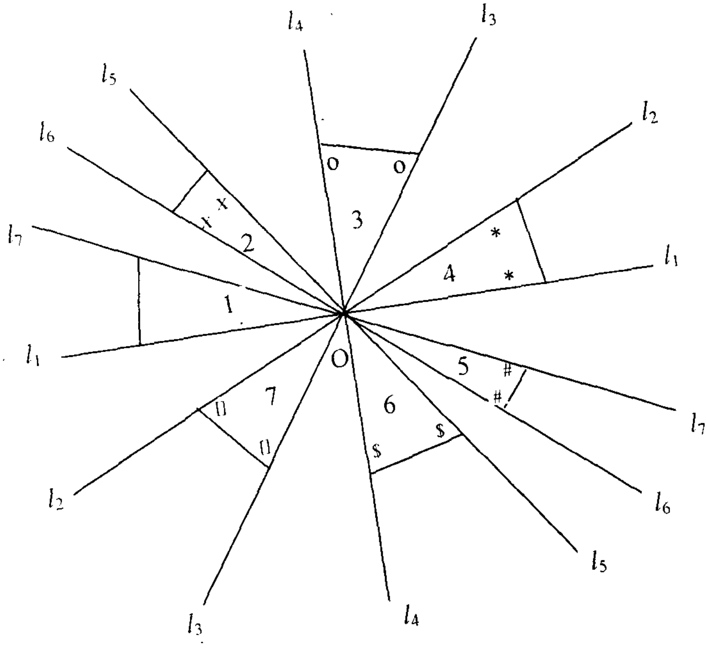
and we get $\boxed{z + t = y} \rightarrow$ (2). (1), (2)

$\Rightarrow t + z + t = 40^\circ + z$

$\Rightarrow 2t = 40^\circ$ (or) $\boxed{t = 20^\circ}$

$\Rightarrow$ **$\angle$ACE = 20°**

56. Seven lines $l_1, l_2, l_3, l_4, l_5, l_6$ and l_7 intersect at O. Find the sum of the base angles of the seven triangles marked therein [shown in the diagram.] [Hint: If two lines intersect the vertically opposite angles are equal.]



Solution: Let the angles formed at vertex O of each of the 7 triangles be $x_1, x_2, x_3, x_4, x_5, x_6, x_7$. By vertically opposite angles, we get another set of angles $x_1, x_2, x_3, x_4, x_5, x_6, x_7$, around O. $\therefore x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = \frac{360^\circ}{2} = 180^\circ$

$\therefore$ Sum of base angles of the seven marked triangles

$$\begin{aligned} &= 7 \times 180^\circ - 180^\circ \\ &= 6 \times 180^\circ = 1080^\circ \end{aligned}$$

57. There are 144 green balls and 216 blue balls. The balls are packed in boxes in such a way that every box contains the same number of balls and every box contains balls of the same colour. What is the minimum number of boxes required? Can you pack the balls in (a) 10 boxes (b) 12 boxes? Given reason for your answer.

Solution: Every box should contain same number of balls. Each box should contain number of balls which is a common factor of 144 and 216.

The common factors of 144, 216 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72. Lesser the number of boxes, more the number of balls in each box (72 balls in each box).

$\therefore$ The minimum number of boxes required

$$= \frac{144}{72} + \frac{216}{72}$$
$$= 2 + 3 = \boxed{5}$$

[i.e. 2 boxes of 72 green balls each, 3 boxes of 72 blue balls each]

It is possible to pack the balls in 10 boxes as follows: 4 boxes of 36 green balls each, 6 boxes of 36 blue balls each.

It is not possible to pack the balls in 12 boxes as 30 is not a common factor of 144, 216.

(i.e. 12 boxes of 30 balls each will contain some boxes of blue and green balls together)

58. The average of 9 distinct natural numbers is 223. (a) Find the biggest of these numbers. (b) Find if these nine natural numbers can be consecutive. If yes find the biggest and smallest of these numbers.

Solution: The sum of 9 distinct natural numbers

$$= 9 \times 223 = \boxed{2007}$$

(a) Among the 9 distinct natural numbers, one number will be the biggest and the other 8 numbers should form the smallest group 1,2,3,4,5,6,7,8.

$\therefore$ The biggest number

$$= 2007 - (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) = \boxed{1971}$$

(b) If 9 numbers are consecutive, the middle number should be equal to the average value 223 .

$\therefore$ The 9 consecutive natural numbers are:

$$219, 220, 221, 222, 223\,, 224, 225, 226, 227.$$

Biggest number $= 227$. Smallest number $= 219$.

59. The sum of 4 three digit numbers is 1545. Find the sum of the numbers if each digit of the above number is replaced by its 8 complement. All the digits of the four numbers lie between 0 and 8. [8 complement of a number 'a is $8 - a$]

Solution: Let the 4 three digit numbers be abc, def, ghi, jkl [abc means 'a' in hundred's place, 'b' in tens place, 'c' in unit's place]. Let 'A' be 8's complement of 'a'. Let 'B' be 8's complement of 'b' and so on till 'L' be 8's complement of 'l'.

We have

$$\begin{array}{rcrcr}
abc & + & ABC & = & 888 \\
+def & + & DEF & = & +888 \\
+ghi & + & GHI & = & +888 \\
\underline{+jkl} & + & \underline{JKL} & = & \underline{+888} \\
\\
\underline{1545} & + & \underline{R} & = & \underline{3552}
\end{array}$$

$$\begin{aligned} \therefore\ R &= 3552 - 1545 \\ &= 2007 \end{aligned}$$

60. Evaluate:

$$\begin{array}{rcccccccccl}
 & & & & & & & & & & \text{There are} \\
 & 27 & 27 & 27 & 27 & & \cdots & 27 & 27 & 27 & \text{[27 twenty sevens]} \\
- & & 27 & 27 & 27 & & \cdots & 27 & 27 & 27 & \text{[26 twenty sevens]} \\
+ & & 26 & 26 & 26 & & \cdots & 26 & 26 & 26 & \text{[26 twenty six's]} \\
- & & & 26 & 26 & & \cdots & 26 & 26 & 26 & \text{[25 twenty six's]} \\
 & & & & \vdots & & & & & & \\
+ & & & & 11 & 11 & \cdots & 11 & 11 & 11 & \text{[11 elevens]} \\
- & & & & & 11 & \cdots & 11 & 11 & 11 & \text{[10 elevens]}
\end{array}$$

Solution:

$$\begin{array}{rcccccccl}
 & 27 & 27 & 27 & 27 & \cdots & 27 & 27 & 27 & \text{[27 twenty sevens]} \\
- & & 27 & 27 & 27 & \cdots & 27 & 27 & 27 & \text{[26 twenty sevens]} \\
\hline
 & 27 & \underline{00} & \underline{00} & \underline{00} & \cdots & \underline{00} & \underline{00} & \underline{00} & \text{[26 sets of } \underline{00}\text{]}
\end{array}$$

$$\begin{array}{rcccccccl}
 & 26 & 26 & 26 & 26 & \cdots & 26 & 26 & 26 & \text{[26 twenty six's]} \\
- & & 26 & 26 & 26 & \cdots & 26 & 26 & 26 & \text{[25 twenty six's]} \\
\hline
 & 26 & \underline{00} & \underline{00} & \underline{00} & \cdots & \underline{00} & \underline{00} & \underline{00} & \text{[25 sets of } \underline{00}\text{]}
\end{array}$$

∴ The result will be

$$\begin{array}{rcccccccc}
 & 27 & 00 & 00 & 00 & \cdots & 00 & 00 & 00 \\
+ & & 26 & 00 & 00 & \cdots & 00 & 00 & 00 \\
+ & & & 25 & 00 & \cdots & 00 & 00 & 00 \\
 & & & & \ddots & \ddots & \ddots & \ddots & \ddots \\
+ & & & & & & 11 & \cdots & 00 \\
\hline
 & 27 & 26 & 25 & & \cdots & 11 & \underline{00} & \cdots\ \underline{00} \\
 & & & & & & & \multicolumn{2}{r}{\text{(Ten sets of } \underline{00}\text{)}}
\end{array}$$

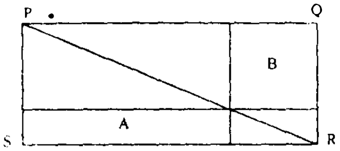
61. Show that $x + (x+1) + (x+2) + \cdots + (x+2006)$ is divisible by 2007. x is a natural number. [Hint $1+2+3+\cdots+n = \frac{n(n+1)}{2}$].

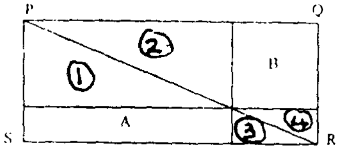
Solution:

$$\begin{aligned} \overbrace{(2007 \text{ times})}\, x + x + x + x + \cdots + x + x &= 2007x \\ 1 + 2 + 3 + 4 + \cdots + 2005 + 2006 &= \frac{2006 \times 2007}{2} \\ &\quad \text{(from Hint)} \end{aligned}$$

Adding, $x+(x+1)+(x+2)+(x+3)+\cdots+(x+2005)+(x+2006)$ $= 2007x + 2007 \times 1003$ which is equal to $2007(x+1003)$ which is always divisible by 2007 for any natural number 'x'.

62. $PQRS$ is a rectangle. PR is its diagonal (see diagram). A and B are also rectangles. Compare the areas of A and B.





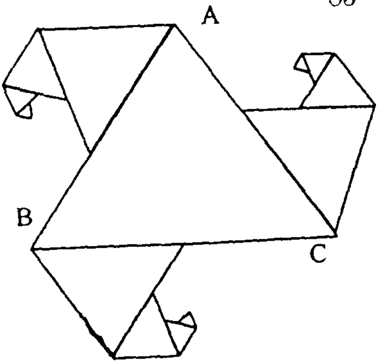
Solution: Diagonal of a rectangle divides it into 2 equal triangular parts.

From the figure, we get

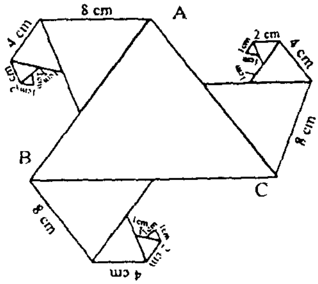
Area (1) = Area (2)
Area (3) = Area (4)

Area (2) + B + Area (4) = Area (1) + A+ Area (3)

$\Rightarrow \boxed{B=A}$. $\therefore$ The areas of rectangles A and B are equal.

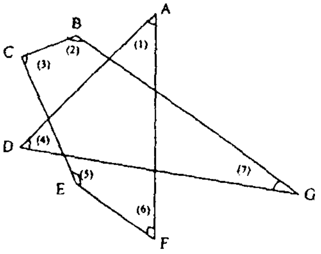


63. *ABC* is an equilateral triangle of side length 16cm. Smaller equilateral triangles are constructed (as shown in the figure), on all the three sides with length of the sides measuring 8cm, 4cm, 2cm, and 1cm. Find the perimeter of this figure.



Solution: The required length of the boundary is clearly three times the length from *B* to *C* along the boundary

$$\therefore \text{Required perimeter} = 3 \times (8 + 4 + 2 + 1 + 1 + 1 + 2 + 4 + 8) \text{ cm}$$
$$= 3 \times 31 \text{ cm} = 93 \text{ cm}.$$

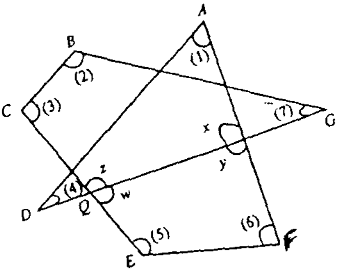


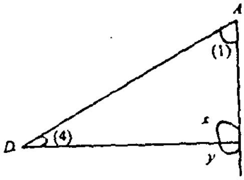
64. In the adjoining figure, find the sum of the angles marked (1), (2), (3), (4), (5), (6) and (7).

Solution: Sum of the angles of a △ is 180°.

Sum of the angles of a quadrilateral is 360°.

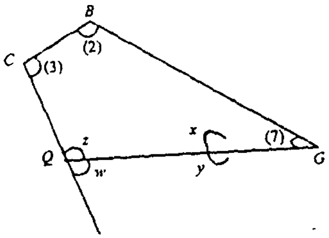
Sum of the linear pair angles is 180°.



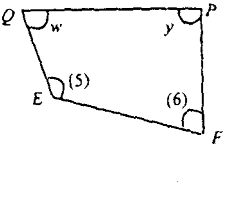


Fig

$$\angle 1 + \angle 4 + x = 180^\circ = y + x \Rightarrow \angle 1 + \angle 4 = y \qquad (1)$$



$$\angle 2 + \angle 3 + \angle 7 + z = 360^\circ = 180^\circ + z + w \Rightarrow \angle 2 + \angle 3 + \angle 7 = 180^\circ + w \qquad (2)$$



$$\angle 5 + \angle 6 + w + y = 360^\circ \qquad (3)$$

$\therefore$ (1) + (2) + (3)

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + w + y$

$= 360^\circ + 180^\circ + w + y$

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle + 5 + \angle 6 + \angle 7 = 540^\circ$

65. Find all positive integers x and y such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{2}$.

Solution: Case I: Both x, y are possible integers

$*$ Both x, y cannot be greater than 4. Otherwise $\frac{1}{x}+\frac{1}{y}<\frac{1}{2}$.

Because $x>4$, $y>4$ $\Rightarrow \frac{1}{x}<\frac{1}{4}, \frac{1}{y}<\frac{1}{4}$. $\therefore \frac{1}{x}+\frac{1}{y}<\frac{1}{2}$.

$*$ x or y cannot be lesser than 3. Otherwise $\frac{1}{x}+\frac{1}{y}>\frac{1}{2}$.

Because x or $y<3 \Rightarrow \frac{1}{x}\geq\frac{1}{2}$ (or) $\frac{1}{y}\geq\frac{1}{2} \Rightarrow \frac{1}{x}+\frac{1}{y}>\frac{1}{2}$.

$\therefore$ One of the integer x (or) y should be 3 or 4.

Let $\boxed{x=3}$. We have $\frac{1}{3}+\frac{1}{y}=\frac{1}{2} \Rightarrow \boxed{y=6}$.

When $\boxed{x=4}$, we have $\frac{1}{4}+\frac{1}{y}=\frac{1}{2} \Rightarrow \boxed{y=4}$.

If $\boxed{y=3}$, then $\boxed{x=6}$.

$\therefore$ There are 3 solutions in this case. They are

(1) $x=3, y=6$ $\quad x=4, y=4$ $\quad x=6, y=3$.

Case II: Both x, y are negative integers

$*$ If x, y are negative, then $\frac{1}{x}, \frac{1}{y}$ are negative

$\therefore$ $\frac{1}{x}+\frac{1}{y}$ will be negative and cannot be equal to $\frac{1}{2}$.

$\therefore$ There is no solution in this case.

Case III: One of the integers x, y is positive and the other is negative

Let $x>0$, $y<0$. Then $\frac{1}{x}>0$. $\frac{1}{y}<0$.

Set

$\frac{1}{x}=m>0$

and $\frac{1}{y} = -n < 0$

where $m, n > 0$.

The equation becomes $m - n = \frac{1}{2}$.

$\therefore\ m > \frac{1}{2} \Rightarrow \frac{1}{x} > \frac{1}{2} \Rightarrow x < 2 \Rightarrow x = 1$ as x is a positive integer.

When $x = 1$, $\frac{1}{1} + \frac{1}{y} = \frac{1}{2} \Rightarrow y = -2$.

Likewise, $y = 1 \Rightarrow x = -2$.

$\therefore$ There are 2 solutions in this case.

They are : (1) $x = 1, y = -2$ (2) $x = -2, y = 1$.

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